RE-ESTIMATION OF LPC COEFFICIENTS IN THE SENSE OF $L_{\infty}$ CRITERION

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ABSTRACT
Now the generally used approaches such as auto-correlation method and covariance method for estimating LPC coefficients are to solve a set of linear equations by using of Levinson-Durbin recursion or lattice formulations, but the LPC coefficients computed are best only in the sense of $L_2$ criterion[1]. For speech processing, $L_{\infty}$ criterion is a more suitable measure metric. The idea of our approach is: from the initial values of LPC coefficients, the residual errors could be reduced step by step by using Least Squares process iteratively until the LPC coefficients are approximately best in the sense of $L_{\infty}$ criterion. Furthermore, this approach could be applied to other problems for estimating some parameters.

1. INTRODUCTION
There are two important methods to extract short-term speech features: one is Linear Predictive (LP) and other is Discrete Fourier Transform (DFT). The DFT is superior to represent the detailed structure of the speech waveform. On the other hand, the LP analysis can well represent the smoothed envelope of the spectrum and rather robust to noise. So most speech recognition systems selected LPC coefficients or the derived ones such as cepstrum or mel-cepstrum, as acoustic features.

Two methods have been used for estimating LPC coefficients, one is the auto-correlation method and the other is covariance method. Both of them have to solve a set of linear equations. Since the linear equations have a special structure, some effective computing algorithms have been developed, such as Levinson-Durbin recursion and lattice formulations, but the LPC coefficients computed are best only in the sense of $L_2$ criterion. For speech processing, $L_{\infty}$ criterion is a more suitable metric[2][3]. Our approach is to refine LPC coefficients in the sense of $L_{\infty}$ criterion. Our experiments showed that the residual errors could be reduced step by step by using Least Squares algorithms iteratively until the LPC coefficients are approximately best in the sense of $L_{\infty}$ criterion. Furthermore, this approach could be applied to other problems for estimating some parameters.

2. LPC ANALYSIS
Let $\{x_i\}$ stand for the speech sample sequence, then the p dimensional LPC analysis is to find a set of coefficients $\{a_i | 1 \leq i \leq p\}$ that can minimize predictive errors in some sense such as $L_2$ criterion. That means the speech sample $x_i$ can be represented by the past p samples with linear relation as,

$$a_1x_{i-1} + \ldots + a_px_{i-p} = x_i + e_i,$$

where $\{e_i\}$ is a probabilistic variable with Gaussian distribution, and its averaged value is zero and its covariance is $\sigma^2$. The predictive errors are required minimum in the sense of $L_{\infty}$ criterion.

Now there are two most widely used methods for estimating the Linear Prediction Coefficients, i.e., Autocorrelation Method and Covariance Method. Note that the above methods can give LP coefficients which are only best or optimal in the sense of least squares metric or $L_2$ measure. We hope to get a set of LP coefficients which can minimize the error in sense of $L_{\infty}$, which can assure the maximum errors of all predictive samples to be minimum.

3. RE-ESTIMATION OF LPC COEFFICIENTS
The traditional methods only consider the overall error of Linear Prediction in $L_2$ metric, but how about the errors of each predicted values and how to control them? We cannot get much information from the above Linear Prediction methods. Our aims are to solve such problems.

For the purpose of following description, let’s introduce some definitions first. Given a speech samples sequence $\{x(n)\}$, and a set of LPC coefficients $\{a_i | 1 \leq i \leq p\}$ in some sense, the predicted error $E_n$ of sample $x(n)$ is defined as follows:
Later we often refer $E_n$ as the error at point $x(n)$. If the errors at some points, such as $E_k$, satisfy the following condition:

$$E_k = \max_n \left| x(n) - \hat{x}(n) \right|$$

Then we refer $E_k$ as the maximum error and $x(k)$ as the maximum error point, or MEP for short. The Linear Prediction error in $L_{\infty}$ metric is defined as follows:

$$E = \min \max_{\{a_i\}} \left| x(n) - \hat{x}(n) \right|$$

where $\hat{x}(n)$ is the predicted value of $x(n)$, and $\{a_i\}$ denote a set of LPC coefficients, the limits of $n$ is within a frame. If a set of LPC coefficients are best in $L_{\infty}$ metric, it must satisfy the above error condition $E$. From the definition, it is clear that the predicted errors at all points would be uniformly minimum in $L_{\infty}$ metric.

Now we present a method to re-estimate LPC coefficients. The idea is as follows: take the original LPC coefficients as initial values, then by reducing the maximum error at MEPs, we can get a new set of LPC coefficients. Take the new LPC coefficients as initial values, repeat this procedure at most $p$ (LPC order) times, we can get LPC coefficients which are best in $L_{\infty}$ metric.

Let $\{a_i^k\}$ denote the LPC coefficients computed after $k$-th iteration procedure, and $E_k^l$ denote the predicted error at point $x(l)$ with respect to the LPC coefficients $\{a_i^k\}$. Let $\{a_i^0\}$ denote the LPC coefficients computed by auto-correlation or covariance method. We now discuss our method in single-MEP case and poly-MEPs case respectively.

### 3.1 Single-MEP Case

In this case, there exists only one MEP. Suppose the MEP is $x(k)$, $\hat{x}(k)$ is the predicted value of $x(k)$, $p$ is the order of Linear Prediction, $\{a_i^0\}$ is the original LPC coefficients, then

$$\hat{x}(k) = a_i^0 x(k-1) + \Lambda + a_p^0 x(k-p)$$

Let $e_k = x(k) - \hat{x}(k)$, then $E_k^l = |e_k|$. Suppose $x(k-l) \neq 0$, and it satisfies the following condition:

$$|x(k-l)| = \max \{|x(k-l)|, \ldots, |x(k-p)|\}$$

Set $\tau x(k-l) = -\text{sgn}(e_k)$ (3.1.1)

where $\text{sgn}(\cdot)$ is a sign function which means:

$$\text{sgn}(x) = \begin{cases} 1 & \text{if } x > 0 \\ -1 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \end{cases}$$

and $\tau$ is a parameter to be determined, here $\tau$ is referred as direction parameter. From formulation (3.1.1) $\tau$ can be computed, $\tau = -\text{sgn}(e_k)/x(k-l)$.

Next we will determine parameter $\lambda$ which is supposed greater than zero and referred as delay parameter. To decrease the maximum predicted error at point $x(k)$, $\lambda$ should meet the following condition:

$$|e_k + \lambda \hat{x}(k-l)| = \max_i |e_i + \lambda \hat{x}(i-l)|$$

Note that the formulation (3.1.1), the above equation can be rearranged as the following form:

$$E_k = \lambda + \max_i |e_i + \lambda \hat{x}(i-l)|$$

Since we have

$$\max_i |e_i + \lambda \hat{x}(i-l)| \leq \max_i E_i + \lambda \max_i |\hat{x}(i-l)|$$

Hence $\lambda$ satisfies the following inequality,

$$\lambda \geq \left[ \frac{E_k - \max_i E_i}{1 + \max_i |\hat{x}(i-l)|} \right]$$

Note that the right of above inequality is greater than zero and smaller than $E_k$, so we choose the lower bound as the value of parameter $\lambda$.

$$\lambda = \left[ \frac{E_k - \max_i E_i}{1 + \max_i |\hat{x}(i-l)|} \right]$$

(3.1.2)

Considering the above formulations (3.1.1) and (3.1.2), the parameters $\tau$ and $\lambda$ have been determined, so we can compute a set of new LPC coefficients by the following steps:

Step 1: For $i=1$ to $p$, but except $l$

$$a_{i}^0 \rightarrow a_{i}^1$$

Step 2: Set

$$a_{i}^0 + \lambda \tau \rightarrow a_{i}^1$$

Then $\{a_i^1\}$ is the new LPC coefficients which can decrease the maximum error at MEP, and the descending magnitude is $\lambda$. The above processing of single-MEP case results in Poly-MEPs.

### 3.2 Poly-MEPs Case

Now let’s consider how to reduce the maximum errors in Poly-MEPs case. Suppose $\{a_i^0\}$ is a set of LPC coefficients, and the corresponding MEPs are $x(i_1), \ldots, x(i_k)$, $1 < k < p$. We hope to find a set of new LPC coefficients $\{a_i^1\}$ which meet the
following conditions:

\[ E^i = \Lambda = E^i = \max_{i \in K \cap \Lambda} \]

According to the Linear Prediction definition, we have the following equations with respect to the MEPs \( x(i_1), \ldots, x(i_k) \),

\[
\begin{cases}
  x(i_1) = a_1x(i_1 - 1) + \Lambda + a_p x(i_1 - p) \\
  \Lambda \Lambda \Lambda \Lambda \Lambda \Lambda \Lambda \Lambda \Lambda \Lambda \Lambda \Lambda \Lambda
\end{cases}
\]

\[
\begin{cases}
  x(i_k) = a_kx(i_k - 1) + \Lambda + a_p x(i_k - p)
\end{cases}
\]

where \( x(i_1), \ldots, x(i_k) \) are predicted values of \( x(i_1), \ldots, x(i_k) \) respectively. We can express this in matrix form:

\[
\mathbf{X} = \mathbf{C}_{kxp} \mathbf{A}
\]

where

\[
\mathbf{X} = \begin{bmatrix}
  x(i_1) \\
  \mathbf{M} \\
  x(i_k)
\end{bmatrix}
\]

\[
\mathbf{A} = \begin{bmatrix}
  a_1 \\
  \mathbf{M} \\
  a_p
\end{bmatrix}
\]

and,

\[
\mathbf{C}_{kxp} = \begin{bmatrix}
  x(i_1 - 1) & x(i_1 - p) \\
  \Lambda & \Lambda & \Lambda \\
  x(i_k - 1) & x(i_k - p)
\end{bmatrix}
\]

Suppose we can choose a sub-matrix \( \mathbf{C}_{kkk} \) from \( \mathbf{C}_{kxp} \) (Note 1<k<p),

\[
\mathbf{C}_{kkk} = \begin{bmatrix}
  x(i_1 - j_1) & x(i_1 - j_k) \\
  \Lambda & \Lambda & \Lambda \\
  x(i_k - j_1) & x(i_k - j_k)
\end{bmatrix}
\]

And the inverse matrix of \( \mathbf{C}_{kkk} \) exists, then we can decrease the maximum errors at MEPs further by method which is analogous to that in single-MEP case.

Let’s first determine the direction vector \( \mathbf{e} \) which is a \( k \)-dimensional vector, and then discuss how to compute the delt parameter \( \lambda \) which is a positive real number.

Considering the sub-matrix \( \mathbf{C}_{kkk} \), \( k \)-dimensional vector \( \mathbf{e} \) can be expressed as follows:

\[
\mathbf{e} = \begin{bmatrix}
  \mathbf{e}_{j_1} \\
  \mathbf{M} \\
  \mathbf{e}_{j_k}
\end{bmatrix}
\]

Another \( k \)-dimensional vector \( \mathbf{S} \) (which is referred as sign vector) is composed of 1 or -1, \( \mathbf{S} \) can be expressed as follows:

\[
\mathbf{S} = \begin{bmatrix}
  -\text{sgn}(e_{i_1}) \\
  \mathbf{M} \\
  -\text{sgn}(e_{i_k})
\end{bmatrix}
\]

Since \( \mathbf{C}_{kkk} \) has an inverse matrix, the direction vector \( \mathbf{e} \) can be computed by the next equation:

\[
\mathbf{C}_{kkk}\mathbf{e} = \mathbf{S} \quad (3.2.1)
\]

As we illustrated above, to obtain direction vector \( \mathbf{e} \), we have to solve a \( k \times k \) equations. Since the order \( k \) of such equations is less than prediction order \( p \) (usually less than 16), the additional computation load is not heavy.

Next we will compute the delt parameter \( \lambda \) which is a positive real number and indicates the error descending magnitude. Let \( K \) denote the number set of \( \{i_1, \ldots, i_k\} \). The principle of determining \( \lambda \) is as following:

\[
\lambda \geq \frac{\max_{n \notin K} e_n - \max_{i \in K} e_i}{1 + \max_{i \in K} \sum_{l=1}^{k} e_{i_l} x(i - j_l)}
\]

So \( \lambda \) must meet follows:

\[
\lambda = \frac{\max_{n \notin K} e_n - \max_{i \in K} e_i}{1 + \max_{i \in K} \sum_{l=1}^{k} e_{i_l} x(i - j_l)} \quad (3.2.2)
\]

Note that the right of above inequality is greater than zero, so we choose the lower bound as the value of \( \lambda \),

\[
\lambda = \frac{\max_{n \notin K} e_n - \max_{i \in K} e_i}{1 + \max_{i \in K} \sum_{l=1}^{k} e_{i_l} x(i - j_l)} \quad (3.2.2)
\]

By the above formulations (3.2.1) and (3.2.2), the direction vector \( \mathbf{e} \) and parameter \( \lambda \) have been determined, now we can compute a set of new LPC coefficients by the following steps,

Step 1: For \( i = 1 \) to \( p \), except that \( i \in K \),

\[
a^{i_1}_i \rightarrow a^{i_1}_i
\]

Step 2: compute vector \( \mathbf{e} \).

Step 3: compute parameter \( \lambda \).

Step 4: For all \( i \notin K \),

Set \( a^{i_1}_i + \lambda \cdot \mathbf{e}_i \rightarrow a^{i_1}_i \)
Then \( \{ a^i, 1 \leq i \leq p \} \) is the new LPC coefficients which can decrease the maximum errors at MEPs, and the descending magnitude is \( \lambda \).

For a given speech sample sequence \( \{x(n)\} \), the re-estimation procedure of LPC coefficients is as follows:

Step 1: compute the LPC coefficients frame by frame in \( L_2 \) metric by auto-correlation or covariance method.

Step 2: for the computed LPC coefficients of a frame, check the MEPs. If only one MEP, apply the algorithm provided in Single-MEP case to re-compute LPC coefficients; if there are at least \( p \) (prediction order) MEPs, output the newest LPC coefficients as required and goto Step 3; otherwise, apply the algorithm provided in Poly-MEPs case to re-compute LPC coefficients.

Step 3: re-compute the LPC coefficients of next frame by Step 2 until all the frames have been processed.

Step 4: output the maximum errors of each frame and program stops.

The above discussion presented the main features and procedures of our approach to re-estimate LPC coefficients in \( L_\infty \) metric.

### 4. EXAMPLES AND RESULTS

To test the effectiveness of the algorithms for re-estimation of LPC coefficients, let’s take an example. For instance, we will compute and reestimate the LPC coefficients of vowel /a/ frame by frame, and compare the predicted errors before and after reestimation.

Main information about vowel /a/: sampling rate is 16KHz, 16 bits, total samples 7397, duration 0.462 second. The frame length is 20ms or 320 samples, so the 7397 samples can be divided into 23 frames (we did not let the frames overlap). The prediction order is 16.

The original speech waveform of /a/ is as follows:

![Figure 4.1 waveform of vowel /a/](image)

By using autocorrelation method, we could compute the LPC coefficients of the first frame of /a/ as:

\[
a^0 = (1.03640, 0.00357, 0.06287, -0.15509, 0.12480, -0.24308, 0.04208, 0.03400, 0.09100, 0.00430, -0.11282, 0.01295, 0.08555, 0.03283, -0.06486, 0.01465).
\]

The corresponding error at MEP is \( E_{\infty} = 0.01630 \).

After the application of Poly-MEPs method, the LPC coefficients of the first frame of /a/ becomes as:

\[
a^1 = (0.07546, -0.18726, 0.41583, 0.00626, -0.15000, -0.43083, 0.16563, 0.46284, 0.12172, 0.18506, -0.12460, 0.07848, -0.31432, 0.34452, 0.26376, 0.01465).
\]

The corresponding error at MEP is \( E_{\infty} = 0.00776 \).

The errors corresponding to the 15 times of LPC coefficients reestimation are as follows respectively:

0.01746, 0.01630, 0.01585, 0.01407, 0.01355, 0.01331, 0.01329, 0.01189, 0.01167, 0.01143, 0.01137, 0.01123, 0.01087, 0.01068, 0.00776.

The above errors could be illustrated by the following figure 4.2. The experiments show that the maximum errors in \( L_\infty \) metric decreased more than 50% after reestimation of LPC coefficients.

![Figure 4.2 Error descending by reestimating of LPC coefficients of a frame](image)

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### REFERENCES

