Flow representation through the glottis having a polygonal boundary shape

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Abstract

This paper presents an analytical approach for representing the flow through the glottis. Based on the polygonal line approximation of the coronal section of the glottis, the velocity field of incompressible, inviscid flow can be conveniently represented using the method of conformal mapping. In this method, the actual physical domain is conformally connected to the canonical domain configured as the infinite strip. Since the general form of analytic potential functions for the uniform flow and the flow due to a point vortex is known in the canonical domain, such flows in the physical domain can be derived by the connecting mapping function specific to the glottal shape. Simulation results were presented to evaluate the influence of a vortex pair placed at the supraglottal section on the streamline and pressure field of the uniform flow.

1. Introduction

To understand the nature of sound source mechanism of speech from the viewpoint of the interaction between the flow and vocal folds, it must be quite important to evaluate the flow through the glottis. It is known that the behavior of flow highly depends on the glottal shape. In addition, vorticity can be generated in the downstream region where the glottis takes a rapidly diverging configuration, thus causing pressure drop across the glottis and forming the loss factor in the one-dimensional equivalent circuit model [1].

To predict the velocity field and vorticity according to the glottal shape and flow characteristics, this paper presents a two-dimensional flow model through the coronal section of the glottis. When the flow is incompressible and inviscid, the two-dimensional flow can generally be represented in a convenient manner using the complex velocity potential [2]. In addition, flow pattern through a bounded region can be represented by the method based on the conformal mapping. By adopting the polygonal approximation of the glottal shape, this physical domain is connected analytically to a canonical domain formed as the infinite strip. In the canonical domain, general form of the potential function is known for several types of flows such as the uniform flow and point vortex, and hence the flow in the physical domain can be derived by the connecting mapping function based on the Schwarz-Christoffel formula [3].

This paper is organized as follows. Section 2 mathematically explains the flow representation model in terms of the complex velocity potential, conformal mapping, and the potential function for the uniform flow and point vortex. Section 3 presents simulation results based on the proposed flow model and Section 4 summarizes our work and gives the conclusions.

2. Representation of the two-dimensional glottal flow

This section describes the method for modeling the two-dimensional glottal flow. The glottal flow typically has a relatively small Mach number if the channel is not so constricted. In addition, the Reynolds number is of the order of 1000 [4]. Therefore, the incompressible and inviscid assumptions can be applied to the present problem. In our model, the boundary shape of the glottis is approximated using polygonal lines as shown in Fig. 1. This physical domain is taken in the extended complex z-plane bounded by lines with vertices \(z_1, \ldots, z_{18}\). The left and right hand sides of the figure connect to the tracheal and pharyngeal regions respectively. On the other hand, an infinite strip domain is illustrated in Fig. 2 as the canonical domain of the conformal mapping.

2.1. Complex variable representation of the two-dimensional flow field

For the two-dimensional incompressible flow, the continuity equation imply that the divergence of the velocity \(v(u, v)\) is zero:

\[
\nabla \cdot v = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0. \tag{1}
\]

If the flow is irrotational, the velocity vector \(v(u, v)\) can be represented using a velocity potential \(\phi\) as

\[
u = \frac{\partial \phi}{\partial y}, \quad v = -\frac{\partial \phi}{\partial x}. \tag{2}\]

Furthermore, there exists a stream function \(\psi\) that satisfies Eq. (1) as

\[
u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}. \tag{3}\]

Equations (2) and (3) yield Cauchy-Riemann relation

\[
\frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y}, \quad \frac{\partial \phi}{\partial y} = -\frac{\partial \psi}{\partial x}. \tag{4}\]
The actual flow field is represented in the physical domain. The boundary shape of the glottis is flexibly approximated by polygonal lines in the $z$-plane.

In the $\zeta$-plane is used as the canonical domain of the conformal mapping.

for the real and imaginary parts of the following complex function

$$f(z) = \phi(x, y) + i\psi(x, y), \quad (5)$$

where $i = \sqrt{-1}$ and $z = x + iy$. Considering Eq.(4), $z$ derivative form of Eq.(5) gives the velocity components $(u, v)$ as

$$\frac{df}{dz} = u - iv. \quad (6)$$

This gives the complex velocity $w = u - iv$. For this reason, $f(z)$ is called the complex potential function.

2.2. Construction of the potential function via the conformal mapping

Eq. (6) indicates that the velocity field in the physical domain can be represented if the potential function $f(z)$ is known. However, it is quite difficult to derive it directly for the given geometrical configuration of the glottis. To solve the problem, it is possible to adopt the method based on the conformal mapping. In this method, the physical domain of the flow field is connected analytically to the canonical domain via the mapping function. For the infinite strip canonical domain shown in Fig. 2, we can derive the potential function explicitly for a number of typical flows as will be discussed in the next section. Thus, the flow pattern in the physical domain can be obtained by these complex potentials and the connecting mapping function.

When the glottal shape is approximated using the polygonal lines, the conformal mapping function can be expressed as follows based on the Schwarz-Christoffel formula:

$$z = g(\zeta) = C \prod_{p=1}^{N} \sinh \frac{\pi}{2} (\zeta - \zeta_p)^{-\beta_p} \frac{\zeta_p}{\zeta}, \quad (7)$$

where $C$ is a complex scaling factor of the mapping, $N$ is the number of vertices, $\beta_p$ is the turning angle at vertex $z_p$, and $\zeta_p$ is the prevertex corresponding to $z_p$ as $\zeta_p = g^{-1}(z_p)$. The location of the prevertex $\zeta_p$ is determined numerically by enforcing conditions on the side length of polygonal lines. Given the potential function $f(\zeta)$ in the canonical domain and the mapping function $z = g(\zeta)$, the complex velocity $w = u - iv$ in the physical domain can be derived as

$$w = \frac{df}{dz} = \frac{df}{d\zeta} \frac{d\zeta}{dz} = \frac{df}{d\zeta} g'(\zeta). \quad (8)$$

2.3. Potential functions in the infinite strip domain

This subsection describes potential functions representing the uniform flow and the flow due to a point vortex in the canonical domain. The potential function is derived so that the boundary condition

$$\nabla \phi(\xi, \eta) \cdot n(\xi, \eta) = 0 \quad (9)$$

is satisfied, where $n(\xi, \eta)$ is a unit vector normal to the boundary of the infinite strip.

2.3.1. Uniform flow

During the phonation, it is assumable that the uniform flow is supplied from the lungs. The analytic function which describes the uniform flow is given as

$$f_u = U\zeta. \quad (10)$$

From $df_u/d\zeta = U$, this function clearly represents the flow $U$ going from left to right along the $\zeta$-axis of Fig. 2. It is easy to confirm that this flow satisfies the boundary condition given in Eq. (9).

2.3.2. Flow due to a point vortex

The glottal flow has a relatively large Reynolds number of the order of 1000. It indicates that the boundary layer on the surface of the glottis is very thin and the bulk of the main flow region is regarded as inviscid. However, in the actual glottal flow, generation of the vorticity due to the boundary layer separation is essential. Therefore, the potential function should be derived for the point vortex when the boundary layer separation is to be represented, for instance, based on the vortex method [5].
Flow due to the point vortex is represented in the canonical domain by the following analytic function

$$f_v = -i \frac{\Gamma}{2\pi} \log \frac{\sinh \frac{\pi}{2}(\zeta - \nu_0)}{\sinh \frac{\pi}{2}(\zeta - \nu_0)}.$$  \hspace{1cm} (11)

The flow from this potential function is irrotational everywhere except at the point where the vorticity is placed (i.e., the circulation $\Gamma$ is concentrated at the point $\zeta = \nu_0$) and satisfies the boundary condition given in Eq. (9). $f_v$ can be derived by considering the point vortex in the upper half plane as follows.

It is easy to show that the domains $S$ and $H^+$ in Fig. 3 are connected conformally by the following mapping function

$$\zeta = \frac{1}{\pi} \log \varpi.$$  \hspace{1cm} (12)

This mapping function is determined so that $\zeta = 0$ corresponds to $\varpi = 1$. Because the conformal mapping keeps the analyticity between both domains, the flow patterns in $S$ and $H^+$ are equivalent when transformed via the above mapping function. Therefore, we first consider the flow due to a point vortex in $H^+$. To satisfy the boundary condition in Eq. (9) along the wall of $H^+$ ($\text{Im} \varpi = 0$), a point vortex at $\varpi = \chi_0$ requires its image at $\varpi = \chi_0$. Therefore, the analytic function is given as

$$f_v = -i \frac{\Gamma}{2\pi} \log \frac{\varpi - \chi_0}{\varpi - \chi_0},$$  \hspace{1cm} (13)

where $\Gamma$ is a parameter representing the circulation of the point vortex. Then the analytic function in $S$ can be obtained by transforming it using the mapping function in Eq. (12) which yields the following relations

$$\varpi = e^{\pi \zeta}, \quad \chi_0 = e^{\pi \nu_0},$$

$$\varpi - \chi_0 = 2e^{\pi(\zeta + \nu_0)/2} \sinh \frac{\pi}{2}(\zeta - \nu_0).$$

Substitution of them into Eq. (13) produces the desired potential function in $S$ as

$$f_v = -i \frac{\Gamma}{2\pi} \log \frac{\sinh \frac{\pi}{2}(\zeta - \nu_0)}{\sinh \frac{\pi}{2}(\zeta - \nu_0)} + C_0,$$  \hspace{1cm} (14)

where $C_0$ is a constant

$$C_0 = i \frac{\Gamma}{4}(\nu_0 - \nu_0).$$

It doesn’t include the complex variable $\zeta$ and hence can be neglected.

### 3. Simulation results

This section presents some simulation results on the glottal flow and the resulting pressure distribution using the proposed method. Here we put a vortex pair at the supraglottal section to examine the influence of them on the uniform flow. As the boundary condition, width of the glottis in Fig. 1 was set at $d = 0.2$ (cm). Expiratory flow rate was set at $U_g = 140$ (cm$^3$/sec) under the assumption that the length of the vocal folds was $l_g = 1.4$ (cm).

#### 3.1. Streamline of the uniform flow with a vortex pair

In the infinite strip domain, the potential function which describes the glottal flow with a vortex pair is represented as
the sum of Eqs. (10) and (11) as

\[ f = U \zeta - i \frac{\Gamma}{2\pi} \log \frac{\sinh \frac{\pi}{2}(\zeta - \nu_0)}{\sinh \frac{\pi}{2}(\zeta - \nu_1)} \sinh \frac{\pi}{2}(\zeta - \nu_2). \]  

(15)

The first term means an uniform flow supplied from the lungs and the second one a vortex pair of opposite circulation. The strength of the uniform flow was determined to conserve the flow rate in the physical domain as

\[ U = \frac{U_g}{l_y}. \]  

(16)

The circulation and the position of the vortex pair were determined arbitrarily in this study.

Fig. 4 shows the flow formed by the potential function in Eq. (15) and the mapping function in Eq. (7). The top and bottom of the figure correspond to the results calculated for the canonical and physical domains, respectively. This experiment simulates the glottal flow just after the separation of the boundary layer and the formation of the vortex pair. It can be seen in the figure that the streamline at the supraglottal section is separated from the glottal wall in the physical domain.

3.2. Pressure drop from the subglottal section

The pressure drop across the glottis is expected to play an important role in the production of voiced sound sources. By assuming the flow to be steady, the pressure difference from the subglottal section \((PD)\) can be calculated by integrating the Bernoulli equation as

\[ PD(g(\zeta)) = \frac{1}{2} \rho \left| \frac{df}{d\zeta} \frac{1}{g'(\zeta)} \right|^2, \]  

(17)

where \( \rho = 1.184 \times 10^{-3} \) (g/cm\(^3\)) is the density of air.

Fig. 5 shows simulation results on the pressure difference across the glottis with (bottom) and without (top) the vortex pair placed at the supraglottal section. The vortex pair induces an increase of the velocity along the center line of the glottis, and hence a decrease of the pressure is caused in the vicinity of them. This result shows that the pressure drop across the glottis can be induced due to vortex-related momentum change.

4. Conclusions

This paper presented a method to represent the flow through the glottal having a polygonal shape. This physical domain was connected to the canonical domain configured as the infinite strip by the mapping function obtained from the Schwarz-Christoffel formula. Potential functions were derived in the canonical domain and they were interpreted in the physical domain to represent the uniform flow and vorticity. Finally, experimental results were presented to simulate the influence of the point vortex on the uniform flow.

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6. References


