FINITE ELEMENT MODEL OF THE HUMAN PHONATION PROCESS

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Abstract

The basis of the human phonation process is given by complex interaction of air flow in the larynx together with structural mechanics of the vocal folds. This paper presents a numerical scheme to model the fluid-solid interaction in the human larynx and its resulting acoustic sound.

The scheme is utilised to simulate the phonation process in a 2D-model. Different geometries of the vocal folds have been used to analyse the effect on the fluid field, the vibration of the vocal folds and the sound generation. The results show the self sustained oscillation of the vocal folds and resolve the Coanda effect.

Keywords: human phonation, fluid-structure interaction, aeroacoustic, finite element method

I Introduction

To simulate the process of human phonation the three physical fields fluid-, solid-mechanics and acoustics are taken into account. Fluid flow describes the airflow through the larynx, which brings the vocal folds to vibrate, and in turn changes the fluid domain. Both, fluid flow and vocal fold vibration, generate sound which propagates through the larynx known as human phonation. The fluid field is modelled with the incompressible Navier-Stokes equations. The solid field is described by the Navier’s equation and the acoustic sound propagation is described by the inhomogeneous wave equation based on Lighthill’s analogy. The coupling between fluid-solid and solid-acoustic is based on continuum mechanics, while the acoustic source term inside the fluid are computed via Lighthill’s analogy. Each of these physical fields is discretised by the finite element method.

Latest finite element laryngeal models have been presented by [9] and [8]. A different approach, based on the immersed boundary method, can be found by [7].

II Methods

In the following, the relevant physical fields for the phonation process and their coupling will shortly be described. The arising partial differential equations (PDEs) are all solved by applying the Finite-Element method (FEM). For a detailed discussion we refer to [5, 6].

II.1 Fluid mechanics

The governing set of partial differential equations for the fluid mechanics is given by the momentum and mass conservation

\[ \rho \frac{\partial \vec{v}}{\partial t} + \rho (\vec{v} \cdot \nabla) \vec{v} + \nabla p - \mu \Delta \vec{v} = 0 , \]

\[ \nabla \cdot \vec{v} = 0 , \]

with \( \vec{v} \) the flow velocity, \( \rho \) the fluid density, \( p \) the hydrodynamic pressure and \( \mu \) the dynamic viscosity.

The equations hold for incompressible fluids which may be assumed due to the fact that for the considered application the Mach number is smaller than 0.3. The computational domain of the fluid flow constantly changes since the vocal folds move and hence define the fluid boundary. The difficulty has been tackled by utilising the Arbitrary-Lagrangian-Eulerian (ALE) approach (for details see [1, 2]).
II.2 Solid mechanics

The mechanical displacement $\bar{u}$ of the vocal folds are modelled by Navier’s equation

$$\nabla \cdot \sigma_s = \rho_s \frac{\partial^2}{\partial t^2} \bar{u}, \quad (3)$$

where $\sigma_s$ denotes the Cauchy stress tensor and $\rho_s$, the density of the solid. Introducing the tensor of elasticity $[c]$ and tensor of linear strain $[S]$, allows us to express Hook’s law by

$$\sigma_s = [c][S] \quad \text{(4)}$$

and the linear strain-displacement by

$$[S] = \nabla \lambda \bar{u} \quad \text{(5)}$$

Substituting (4) and (5) into (3) results in the final PDE for linear elasticity

$$\mathcal{B}^T [c] \mathcal{B} \bar{u} = \rho_s \frac{\partial^2}{\partial t^2} \bar{u} \quad \text{(6)}$$

with the differential operator $\mathcal{B}$ (here given explicitly for the 2D plane case

$$\mathcal{B} = \begin{pmatrix} \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial y} \end{pmatrix} \quad \text{(7)}$$

II.3 Fluid-solid interaction

The air and vocal folds share a common interface $\Gamma_{fs}$ so that the nodes for both fields must coincide, given by

$$\bar{x}_f = \bar{x}_s \quad \text{on } \Gamma_{fs} \quad \text{(8)}$$

Fluid velocity and the first time derivative of the solid displacement are identical since the fluid adheres at the body resulting in the following condition

$$\bar{v} = \frac{\partial}{\partial t} \bar{u} \quad \text{on } \Gamma_{fs} \quad \text{(9)}$$

This implies for solid mechanics the following inhomogeneous Neumann boundary condition

$$[\sigma_s] \cdot \bar{n} = [\sigma_i] \cdot \bar{n} \quad \text{on } \Gamma_{fs} \quad \text{(10)}$$

describing the equivalent of fluid stress $[\sigma_i]$ and solid stress $[\sigma_s]$ in normal direction $\bar{n}$. The fluid stresses can be written explicitly by the hydrodynamic pressure $p$

and fluid velocity $\bar{v}$ as

$$\bar{\sigma}_s = \rho_f \int_{\Gamma_{fs}} -p \cdot \bar{n} \, dx \quad \text{pressure} \quad \text{(11)}$$

$$+ \int_{\Gamma_{fs}} \mu (\nabla \bar{v} + (\nabla \bar{v})^T \cdot \bar{n}) \, dx \quad \text{shear} \quad \text{(12)}$$

Having Dirichlet boundary condition for the fluid and Neumann boundary conditions for solid mechanics, the fluid-solid interaction is also called Dirichlet-to-Neumann problem.

II.4 Acoustic field

As a basis taking the equation of continuity and momentum, Lighthill’s equation in pressure form is derived (for details see [4])

$$\frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} - \Delta p' = \nabla \cdot (\nabla \cdot T) \quad \text{(13)}$$

with $c$ is the speed of sound and $T$ the Lighthill tensor

$$T_{ij} = \underbrace{\rho_f v_i v_j}_{\text{Reynolds stress}} + \underbrace{\tau_{ij}}_{\text{Viscous stress}} + \underbrace{[(p - p_0) - c^2(p - p_0)]\delta_{ij}}_{\text{Heat conduction}} \quad \text{(14)}$$

Thereby, $p_0$ denotes the mean pressure, $\rho_f$ the fluid density and $\rho_0$ its mean density. Viscous stress may be neglected [4] and the heat conduction is assumed to be zero, which leads to the following approximation of (15)

$$T_{ij} \approx \rho_f v_i v_j \quad \text{(16)}$$

The oscillation of the vocal folds induce sound, which is a surface coupled phenomenon. Along the moving boundary $\Gamma_{fs}$ the following relation for the mechanical surface and the acoustic pressure needs to be fulfilled

$$\frac{\partial}{\partial t} \bar{u} \cdot \bar{n} = \bar{v}_a \cdot \bar{n} \quad \text{on } \Gamma_{fs} \quad \text{(17)}$$

Condition (17) forces that the acoustic particle velocity $\bar{v}_a$ are identical to the surface velocity in normal direction. For the considered case it is assumed, that there is no back reaction of the acoustic onto the solid. Using the linearised Euler equation

$$\frac{\partial}{\partial t} \bar{v}_a \cdot \bar{n} = -\frac{1}{\rho_f \partial n} p' \quad \text{(18)}$$
the source term in acoustic pressure formulation is
\[
\frac{\partial}{\partial t} p' = -\rho_f \frac{\partial^2}{\partial t^2} \mathbf{u} \cdot \mathbf{n} \quad \text{on } \Gamma_{fs}. \tag{19}
\]
For the FE formulation and its verification we refer to [3].

Results

The geometric setup of the vocal folds have been adopted from the model presented in [8] and inserted into our computational domain. A fluid pressure condition is given at inflow and outflow of the domain. The resulting simulations shows the development of the Coanda effect - the air jet at the glottis randomly attaches to either side of the trachea wall, as shown in Fig. 1. Furthermore, the occurring fluid flow forces realistic self-sustained vocal fold oscillation. The vibrations of the vocal folds have been analysed and show for different forms different frequencies in their movement. An eigenfrequency analysis shows that the vibrational frequency correlates with the first eigenfrequency of the vocal folds. The generated sound showed dominant peaks in the frequency domain, which vary with the geometric form of the vocal folds.

Conclusion

A computational scheme has been presented to simulate the human phonation process with all relevant physical fields. To the author’s best knowledge this fully coupled scheme is novel. The model is applicable for a parameter study, to analyse the effect of different forms of vocal folds and different pressure conditions for in and outflow on the acoustic sound.