

# Holographic Technique for Linguistic Modelling

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## Abstract

An approach to implement the idea of linguistic modelling by Fourier-holography technique is proposed. The approach is based on deep analogies between Optics as a physical reality and Fuzzy Sets Theory as an abstract construction. We use the concept of linguistic variable, proposed by L.Zadeh and restrict our consideration to the semantics only. We develop a model of 4-f Fourier-holography set-up and demonstrate Fourier-duality of conjunction and disjunction leads to fuzzy-valued logic. We propose biologically inspired approach to implement multi-input reasoning in the set-up and demonstrate experimental illustration by using classical example of General Modus Ponens rule.

## 1. Introduction

Long time Artificial Intelligence Systems (AI) were designed and developed on the base of strong logic as a software and traditional computer as hardware. Now it is acknowledged, this approach doesn't allow a number of real AI tasks to be solved successfully. The cause for this problem is the strong logic, which was defined by Aristotle and was a foundation for European scientific paradigm for 20 centuries, doesn't allow human way of thinking to be described adequately.

The concept of linguistic modelling was introduced by L.Zadeh to mimic human way of reasoning [1]. The concept allows human-machine interface to be organized by "natural-like" language using, e.g. it can be used by human-teacher to describe for machine the way of the problem solving, which is based on human's knowledge and experience. Such imprecision words as "big", "very big", "not very big", etc can be used for human's knowledge representation. Technically, the problem is to match metric scale, used by the technical device, with the linguistic scale, intuitively formed by the person.

Mathematically, linguistic modelling is based on Fuzzy Set Theory (FST). From implementation "in hardware" point of view main problem is the approach is computationally expensive. To solve this problems optics gives attractive alternative to their electronic counterpart due to the ability of optical devices to represent, percept and process images as 2-D information in parallel. In addition to this natural parallelism of optical information processing, a number of deep analogies between optical holography and main attributes of both neural networks paradigm and FST have to be mentioned:

- global connectionism due to the diffraction phenomenon and free-space interconnections;
- learnability due to the holographic technique usage;
- associatability as a property of hologram;
- the fact that the inter-connectedness of parallel optical signals reflects inner-connectedness of real-life information;
- basic phenomenon of diffraction is the main source of "fuzziness" in optics;
- basic phenomenon of wave interference is a mechanism for fuzzy relation forming;
- finally, the diffraction of a wave on a hologram is similar to a mapping on the fuzzy relation.

These analogies between main principles of both NN Paradigm and FST from the one hand, and attributes of optical holography from the other hand, determine our interest in optical realisation of neuro-fuzzy systems.

Our main interest in this paper is to develop the linguistic model that corresponds to the Fourier-holography setup. In this paper we restrict our consideration to the semantics only. We develop an algebraic description of 4-f Fourier-holography set-up by using triangular norms based approach. In the model we use the Fourier-duality of the t-norms and t-co-norms, which is implemented by 4-f Fourier-holography set-up. We demonstrate the set-up is described adequately by De-Morgan's law for involution. Fourier-duality of the t-norms and t-conorms leads to fuzzy-valued logic. We consider General Modus Ponens rule implementation to define the semantical operators, which are adequate to the setup. We consider scales, formed in both +1 and -1 orders of diffraction. We use representation of linguistic labels by fuzzy numbers to form the scale and discuss the dependence of the scale grading on the holographic recording medium operator. Then we propose a biologically inspired approach to implement multi-input reasoning. We present experimental realisation of General Modus Ponens rule.

## 2. Model

### 2.1. Used definitions and approach

Definition 2.1. Following by Zadeh [1], let us define linguistic variable as a set  $\langle Y, Tm(Y), U, G, M \rangle$ , where  $Y$  is a name of the variable,  $Tm(Y)$  – a term-set,  $U$  – an universal set,  $G$  – the syntax, which generates the terms of set  $Tm(Y)$ ,  $M$  – the semantics, which generates a sense  $M(Y)$  for each linguistic value  $Y$ . In the paper we consider semantics  $M$  only.

**Definition 2.2.** Let us define logic as an algebraic construction  $\langle U, D, \wedge, \vee, M \rangle$ , where  $\wedge$  and  $\vee$  are conjunction and disjunction, respectively,  $D$  is an operator of duality.  $D$  is as a mapping  $D: [0,1] \rightarrow [0,1]$ , defined by axioms

$$D(1) = 0, D(0) = 1, \quad (2.1)$$

$$D(a) \leq D(b) \text{ if } a \geq b \text{ for all } a, b \in [0,1].$$

(2.2)

Let  $M$  satisfies the lattice conditions, i.e.  $\forall x, y \in Tm(Y)$

$$M(x \wedge y) \leq M(x) \wedge M(y)$$

$$M(x \vee y) \geq M(x) \vee M(y)$$

**Definition 2.3.** Let  $V: [0,1]^2 \rightarrow [0,1]$  be a commutative, associative, non-decreasing binary operation with neutral element  $e$ , i.e.  $V(a, e) = V(e, a) = a$  for all  $a \in [0,1]$ . Then

(i) if  $e = 1$ ,  $V$  is called a conjunction ( $V = \wedge$ ),

(ii) if  $e = 0$ ,  $V$  is called a disjunction ( $V = \vee$ ).

Let us use a general De Morgan's law relating conjunctions and disjunctions in the form for involution

$$(a \wedge b) = D((D(a) \vee D(b)))$$

$$(a \vee b) = D((D(a) \wedge D(b))).$$

(2.3)

**Definition 2.4.** Let an unlimited plane wave be universe  $U$ . Then any image  $Im$ , i.e. optical field in any plane, or a transparency, is a subset of  $U$ . Pixels are members and membership function is  $Im: U \rightarrow [0,1] \times P_{Im}$ , where  $[0,1]$  represents an interval of normalised amplitudes,  $P_{Im}$  represents a phase term. For simplicity, but without the loss of generality, we consider  $Im$  as amplitude one only, i.e.  $P_{Im} = 0$ .

**Definition 2.5.** Let the sense  $M(Y)$  of linguistic value  $Y$  be represented by an image  $Im_y$ . Let us note, we don't restrict the images to fuzzy numbers.

## 2.2. Logical description

Let us consider 4-f Fourier-holography setup with plane reference wave, presented in Fig.1..

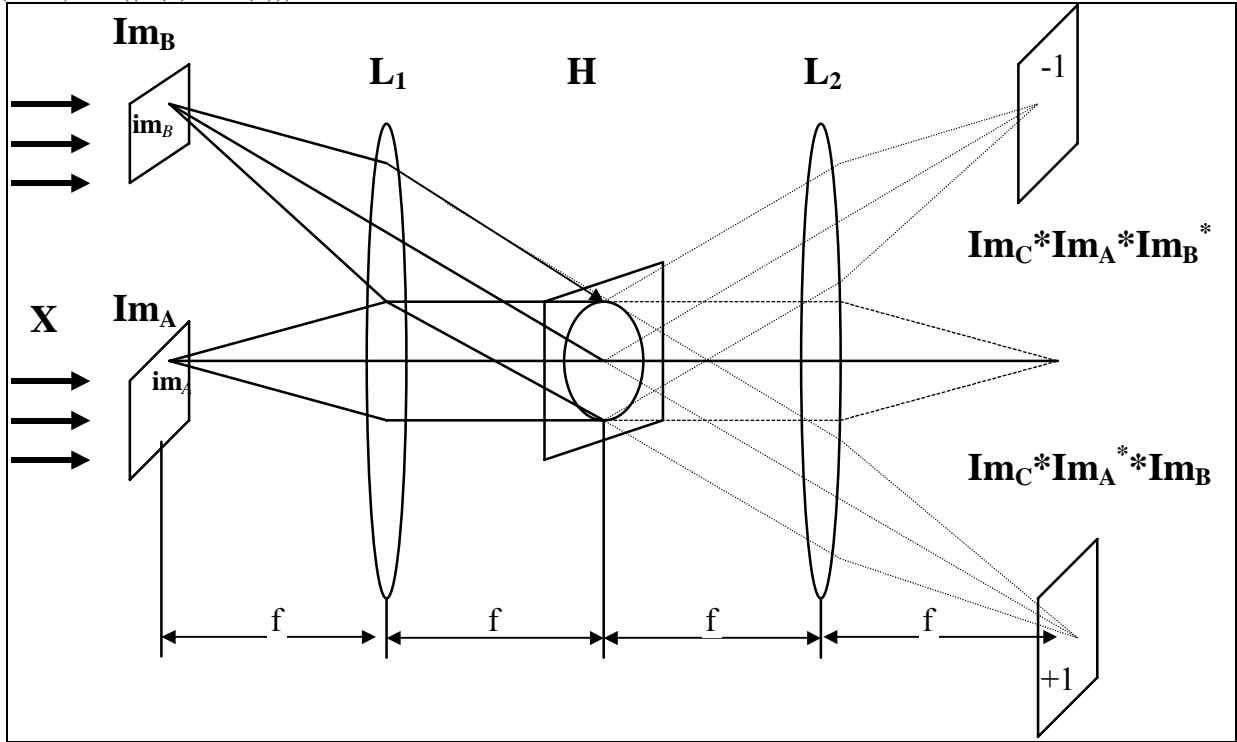


Figure 1. 4-f Fourier-holography set-up.

Conjunction in optics is implemented by positive transparency illumination [2]. If the transparency is prepared by conventional two-stage negative-positive photo-recording process, then the conjunction is described by

$$(Im_A \wedge Im_B) = N(N(Im_A)) \cdot N(N(Im_B)).$$

Usually, transmittance of negative recording medium is normalised by conditions  $\tau_{max}=1, \tau_{min}=0, Im_{max}=1, Im_{min}=0$ . However, this procedure doesn't allow the condition for neutral element to be met. If the negative recording media operator is not involution, i.e.  $N(N(Im_A)) \neq Im_A$ , then the lattice condition is met by the

conjunction in limited area  $[Im_{A \max}(Im_B), Im_{B \max}(Im_A)]$ , determined by  $\forall Im_A, Im_B \in [0,1], Im_A > Im_B : Im_{A \max}(Im_B) \leq N(N(Im_A)) \wedge N(N(Im_B))$ .

In our previous papers it was demonstrated, 4-f Fourier-holography set-up implements De Morgan law (2.3) for Fourier-duality. It leads to the disjunction, which is implemented by Fourier-holography technique

$$(Im_A \vee Im_B)_F = F(F(Im_B) \cdot \eta(F(Im_A))), \quad (2.3)$$

where  $\eta$  is holographic recording media operator. Let us note, De Morgan laws were proposed for negation initially, we apply it to Fourier-duality. To confirm

correctness of this application mathematically, in paper [3] we used an apparatus of additive generators.

It's easily to see from (2.3) neutral element for conjunction is implemented by point-source, described by delta-function.

To define the operator  $M$ , which is adequate to the set-up, we consider an inference in form of Modus Ponens rule

Implication: if  $C$  is  $Im_A$ , then  $B$  is  $Im_B$

Premise:  $C$  is  $Im_C = Im_A'$

Conclusion:  $B$  is  $Im_B'$

where  $A$ ,  $B$ , and  $C$  are linguistic variables, and  $Im_A$ ,  $Im_A'$ ,  $Im_B$ ,  $Im_B'$ , and  $Im_C$  are fuzzy sets representing linguistic labels. This inference can be implemented by the setup in both  $-1$  and  $+1$  orders of diffraction, i.e. in the terms of logical operators

$$((Im_A' \vee M(A \rightarrow B)) = Im_A' * F(\eta(F^*(Im_B)F(Im_A)))$$

for  $-1$  diffraction order, and

$$((Im_A' \vee M(A \rightarrow B)) = Im_A' * F(\eta(F(Im_B)F^*(Im_A)))$$

for  $-1$  diffraction order, where symbol  $\rightarrow$  denotes implication operator, asterisk – complex conjugation, and  $*$  - convolution. Thus, operator  $M$  is defined for  $-1$  order of diffraction by the expression

$$M(A \rightarrow B) = F(\eta(F(Im_A)F^*(Im_B))), \quad (2.4.a)$$

and for  $+1$  order of diffraction by the expression

$$M(A \rightarrow B) = F(\eta(F^*(Im_A)F(Im_B))). \quad (2.4.b)$$

### 2.3. Multi input: an approach

This result allows classical approach to implementation of linguistic modeling by using fuzzy numbers to represent the sense of linguistic variables to be implemented in both  $-1$  and  $+1$  orders of diffraction if one input variable is connected with one output one. However, in real tasks, e.g., in the task of medical diagnostics, a car driving, etc., we have to make an integral conclusion on the base of a number of input variables. It can be represented in a form of General Modus Ponens rule:

Implication: if  $C$  is  $Im_{A1}$ , and  $Im_{A2}$ , and  $Im_{A1}, \dots$  and  $Im_{AN}$ , then  $B$  is  $Im_B$

Premise:  $C$  is  $Im_C = Im'_{A1}$ , and  $Im'_{A2}$ , and  $Im'_{A1}, \dots$  and  $Im_{AN}'$

Conclusion:  $B$  is  $Im_B'$

Semantical operator (2.4.a), traditionally used in the task, doesn't allow us to solve the problem. The problem can be solved by using the operator (2.4.b), however, this operator is a subtraction, and therefore the system cannot evaluate the variables, which have smaller meanings, than the reference one. The scale is formed by learning procedure, i.e., by the hologram recording; two gradations: additive zero and reference meaning are submitted to form the scale. Thus, the system will not sense the meanings, which are worse, than the reference one. To avoid this problem the scale has to be formed by two gradations: additive zero and worse meaning of the linguistic variable.

It is acknowledged, our brain thinks not by numbers, but by patterns. Therefore, the idea to represent meanings of the information under processing by

numbers (including fuzzy numbers), does not correspond to the biological prototypes of AI systems. It is known, the pattern of neural activity of a brain has chaotic structure if there is no perceptible information. If the information is recognized, then the length of correlation is increased.

To combine these results of neuro-sciences within the approach, we use a realization of Fractal Brownian Motion (FBD) as the pattern, which represents input information. Significance of each linguistic variable is represented by the square of the area in the pattern, assigned to this variable. Meaning of a variable is described by a fuzzy number and linked with the Fourier-transform of the pattern's corresponding area by the expression

$$Re(F(Im_i)) = Re(F(N_i)), \quad (2.5)$$

where  $Im_i$  is the area of the pattern, that is assigned to the variable,  $N_i$  – fuzzy number, representing the meaning of the linguistic variable.

## 3. Experimental illustration

### 3.1. One input – one output

Let us consider a classical example of inference "General Modus Ponens":

"If an apple is red, then the apple is ripe".

Two linguistic variables are used here: redness and ripeness. Classical approach is to represent the meanings by fuzzy numbers [1]. The task of the system's learning is to match metric scales of the machine and linguistic scales of the human-teacher. Following to the classical approach, we represent the reference redness by an image, which corresponds to the conditions on fuzzy numbers and therefore, the image is fuzzy number 1. As it follows from the consideration of arithmetic of fuzzy numbers (FN) holographic realization [2], the reference ripeness is represented by FN 2 in  $-1$  diffraction order of the set-up. Not enough ripeness of apples be represented by FNs from interval of fuzzy numbers [1,2], super ripeness - by FNs  $> 2$ .

### 3.2. Tuning of the scales

Operator  $\eta$  in (2.4) determines a number of grades  $U/\delta$  in the scale  $[FN1, U]$ , and allows the logic to be tuned at various range of the scale  $[FN1, FN2]$  and  $[FN2, U]$ . The first range corresponds to "not enough ripeness", the last one corresponds to "too ripe apples". Operator  $\eta$  determines:

- ◆ possible amount of gradation  $X/\delta$  of a scale  $[\delta, X]$
- ◆ reallocation of gradation of a scale between intervals  $[1,2]$  and  $[2, X]$  by means of the determination 1. At increase of parameter  $(X/1)$ , the logic within the framework of an example "feels" a degree of over-ripeness better. On the contrary, at reduction this parameter, the logic becomes "more intelligible" in an estimation of a degree unripeness.

To illustrate this effect, we represent in Fig.2. a Fourier-transform of fuzzy number 1, that represents meaning “red” (curve A), and sections of two holograms of this Fourier-transform: hologram B was recorded with non-linear operator of holographic recording medium  $\eta$ , and hologram C was recorded with linear one.

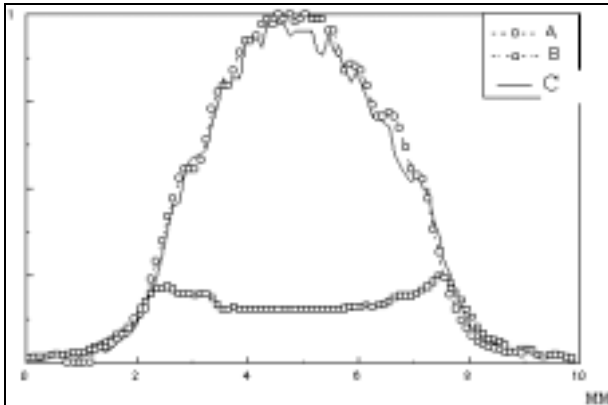


Figure 2. A – Fourier-transform of fuzzy number 1, that represents “red”, and sections of holograms, recorded with non-linear (B) and linear (C) operators of holographic recording medium .

In Fig.3 we represent fuzzy number 1 (A), and fuzzy numbers 2, restored by holograms B and C, correspondingly.

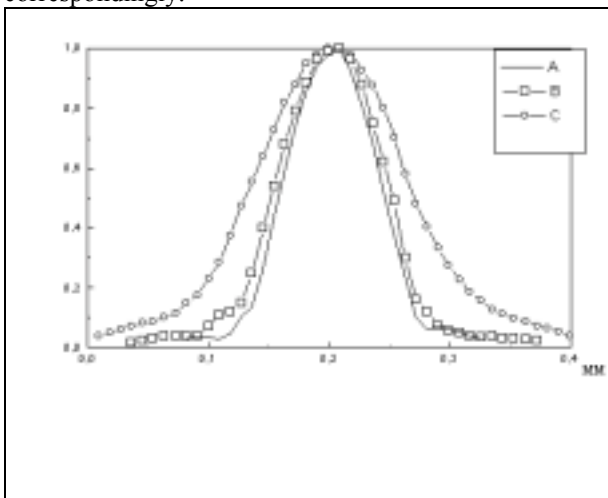


Figure 3. A – fuzzy number 1, B and C – fuzzy numbers 2, restored by holograms B and C, correspondingly.

Table 1.

	Color		Mellowness		Size		Taste		Output
	Subjective evaluation	Procedure	Subjective evaluation	Procedure	Subjective evaluation	Procedure	Subjective evaluation	Procedure	Subjective evaluation by the expert
$Im_1$	Green	Sharpening	Dry	Sharpening	Small	Sharpening	Tasteless	Sharpening	Dis-gusting
$Im_2$	Red	-	Dry	Sharpening	Small	Sharpening	Very taste	Smoothing	So-so
$Im_3$	Very red	Smoothing	Dryish	Sharpening	Very big	Smoothing	Tasteless	Sharpening	Bad
$Im_4$ (Ref.)	Red	-	Juicy	-	Big	-	Taste	-	Good
$Im_5$	Very red	Smoothing	Very juicy	Smoothing	Very big	Smoothing	Very taste	Smoothing	Very good

Operator B ensures the greater number of gradation in a range of scale  $[2, X]$ , i.e. adjusts logic on large "intelligibility" in a degree over-ripeness at the expense of lowering "intelligibility" in area unripeness,

Operator C - reallocates "intelligibility" for the benefit of unripeness, as ensures the greater number of gradation in range  $[1,2]$ . The greater effect of set-up of logic on this or that scale band can achieve at usage of the holographic registering medium with more expressed nonlinear responses, than for the medium, used in experiment, PFG-03M with standard developing process GP-8.

### 3.3. Multi input – one output

An example of Modus Ponens rule “If an apple is red, juicy, big, and tasty, then the apple is good” was implemented. Fractal Brownian Motion  $1024 \times 1024$  with Hurst parameter  $H=0.1$  was used to represent 4 input linguistic variables. An area of the image of FBD was assigned to each linguistic variable to represent the meaning. A square of the area represented subjective evaluation of the variable significance in the integral evaluation of an apple. Five examples with different properties, described in Table 1, were prepared. In accordance with (2.5), if the evaluation of a variable was improved, then the image in the corresponding area of the FBD was smoothed, and v.v., if the evaluation become worse, then the image in the area was sharpened.

The worse image  $Im_1$  was used as the reference one to record three holograms with different operators of holographic recording media. The sections of the holograms are represented in Fig.4.

0.5-cuts of the responses in +1 order of diffraction of the set-up for each apple  $Im_1 - Im_5$ , formed by each hologram, are presented in Fig.5. Horizontal axis here represents subjective evaluation of the apples by the expert, vertical axis is a metric scale of the device.

Thus, these three curves demonstrate such attribute of human thinking as subjectivity. Each hologram forms individual matching of linguistic scale and metric scale, exactly as each person has subjective point of view on the apple – one person can evaluate an apple as very good apple, another person can evaluate the same apple as very bad one.

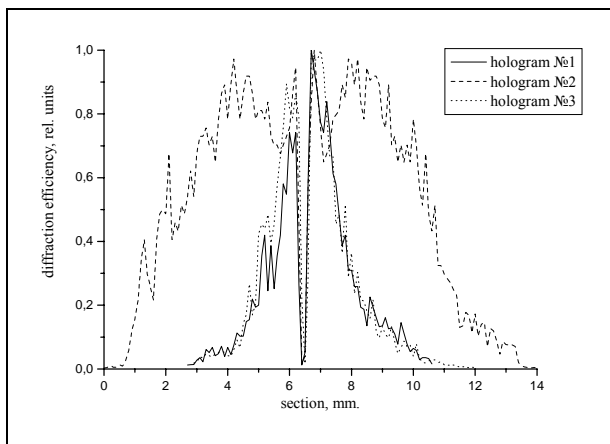


Figure 4. Sections of the holograms.

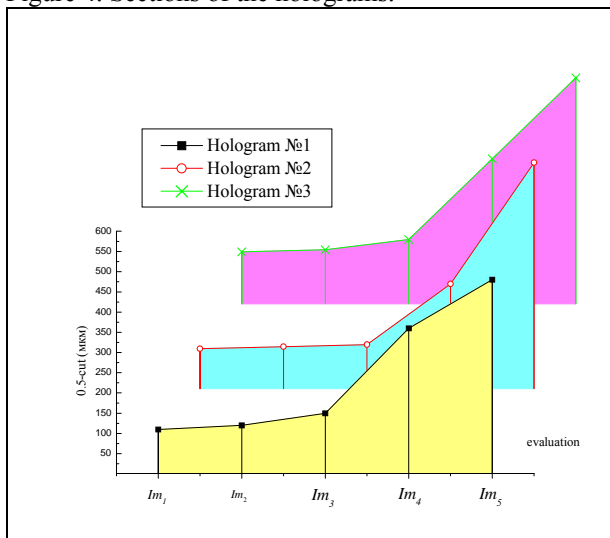


Figure 5. 0.5 cuts of the holograms responses.

#### 4. Conclusion

Thus, Fourier-holography technique can be used to implement the idea of linguistic modelling due to Fourier-duality of conjunction and disjunction, implemented in this set-up in a form of De-Morgan's law. The system is model-free, learning is used to match metric scale of the system with the linguistic scale of a teacher. The learning is implemented in a form of hologram recording by presenting two reference points of the scale. Such attribute of human thinking as subjectivity of evaluation is implemented by using non-linear operator of holographic recording media. To implement the idea of integral evaluation of a multi-input, we implemented such attribute of human thinking

as thinking by patterns. We used four input linguistic variables for simplicity; both: the approach and holography technique allow the number of the variables to be increased.

Our future aim is to implement such attribute of human thinking as context-dependence of thinking.

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#### 5. References

- [1] L.A. Zadeh Information Sciences, 8. 199, 301, 43, (1975).
- [2] A. V. Pavlov: An algebraic model for application of optical neuro-fuzzy systems to networking problems, Soft Computing, v. 5 (2001), №4, p. 318-324.
- [3] A.V. Pavlov Using the methods of Fourier holography to construct logic processors, Journal of Optical technology, v.69, i.10 (2002, October) pp. 735-740
- [4] A.V.Pavlov, "Optical holography and computational intelligence: algebraic foundations", Proc. of Applications and Science of Neural Networks, Fuzzy Systems, and Evolutionary Computations (45th SPIE Annual Meeting), SPIE Proc., Vol.4120, pp.238-245.