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BOUND FOR MINKOWSKI METRIC BASED ON L_p DISTORTION MEASURE

J. S. Pan†‡, F. R. McInnes†and M. A. Jack†
e-mail: jspan@ccir.ed.ac.uk
†Centre for Communication Interface Research, University of Edinburgh, UK
‡Department of Electronic Engineering, Kaohsiung Institute of Technology, Taiwan

ABSTRACT

A bound for Minkowski metric based on L_p distortion measure is proposed. This bound provides a better criterion than the absolute error inequality (AEI) elimination rule on the Euclidean distortion measure. For the Minkowski metric of order n, this bound contributes the elimination criterion from L_1 metric to L_n metric.

1 INTRODUCTION

Vector Quantization (VQ) [1] has been widely used for various applications involving VQ-based encoding and VQ-based recognition. The response time of encoding and recognition is a very important factor to be considered for real-time application. Unfortunately, a full search algorithm is applied in VQ encoding and recognition and it is a time consuming process when the codebook size is large. A vector quantizer of rate R bits/sample and dimension k is a mapping from a k-dimensional vector space into some finite subset $C = \{C_j; j = 1, ..., N\}$, where $N = 2^{kR}$ [2]. The subset C is called a codebook and its elements C_j are called codewords, reproducing vectors, prototypes, or design samples. A distortion measure $D(X, C_j)$ is a nonnegative dissimilarity measure between vector X and codewords C_i. This distortion is used to measure how close the input vector X to these codewords C_j. The nearest codeword is to be selected in order to encode the input vector X. Therefore, encoding each input vector requires N distortion computations and $N - 1$ comparisons.

The codeword searching problem in vector quantization is to assign one codeword to the test vector in which the distortion between this codeword and the test vector is the smallest among all codewords. Given one codeword C_t and the test vector X in the k dimensional space, the distortion of the square Euclidean metric can be expressed as follows:

$$D(X, C_t) = \sum_{i=1}^{k} (x^i - c_t^i)^2, \qquad (1)$$

where $C_t = \{c_t^1, c_t^2, ..., c_t^k\}$ and $X = \{x^1, x^2, ..., x^k\}$.

There requires k multiplications and $2k - 1$ additions for each distortion calculation. Therefore, we need to perform $k2^{kR}$ multiplications, $(2k - 1)2^{kR}$ additions, and $2^{kR} - 1$ comparisons for encoding each input vector. Obviously, the computation complexity depends on codebook size and dimensions. It needs large codebook size and higher dimension for high performance in VQ encoding and recognition systems. It will cause much computation time during codeword searching.

The partial distortion search (PDS) [3] is a simple and efficient codeword searching algorithm which has no extra storage or preprocessing requirements. Given the current minimum distortion,

$$D(X, C_t) = D_{min}, \qquad (2)$$

$$\text{if} \quad \sum_{i=1}^{s} (x^i - c_j^i)^2 \geq D_{min}, \qquad (3)$$

$$\text{then} \quad D(X, C_j) \geq D(X, C_t), \qquad (4)$$

where $s \leq k$.

The efficiency of PDS derives from elimination of an unfinished distortion computation if its partial accumulated distortion is larger than the current minimum distortion. PDS can be further improved based on the computer architecture [4][5].

The hypercube approach is quite efficient if the difference for any coefficient is generally larger than the difference of the other coefficients, such as the first coefficient of cepstrum coefficients. Assume Eq. 2 has already existed,

$$\text{if} \quad |x^i - c_j^i| \geq \sqrt{D_{min}}, \qquad 1 \leq i \leq k, \quad (5)$$

then C_j will not be the nearest neighbour to X.
There is no multiplication operation required for the test of the hypercube approach.

The absolute error inequality (AEI) [6] is the mathematical relationship between the city block metric (or L_1) and the Euclidean metric (or L_2). Assume C_t is the current nearest neighbour to X,

$$\text{if} \quad \sum_{i=1}^{s} |x^i - c_j^i| \geq \sqrt{kD_{min}}, \qquad (6)$$

$$then \qquad \sum_{i=1}^{k}(x^i - c_j^i)^2 \geq D_{min}, \qquad (7)$$

where $s \leq k$.

This means C_j will not be the nearest neighbour to X if Eq. 6 is satisfied. Set $s = 1$ and compare Eq. 5 and Eq. 6. Obviously, the hypercube approach provides a tighter bound than AEI for $s = 1$. This gives us the motivation to investigate the AEI and extend the Euclidean metric to the Minkowski metric of order n.

In section 2, the bound for Minkowski metric is derived and also list some properties of this bound. The application of this bound to fast codeword searching algorithm for Euclidean metric is presented in section 3. The experimental results of minimax method, minimax method including AEI criterion and this new fast codeword searching algorithm for Euclidean metric are shown in section 4. Finally, the conclusions of this new bound are made in the last section.

2 BOUND FOR MINKOWSKI METRIC

Given one codeword C_t and the test vector X in the k dimensional space, the distortion of the Minkowski metric of order n can be expressed as follows:

$$D_{min} = D(X, C_t) = \sum_{i=1}^{k} |x^i - c_t^i|^n \qquad (8)$$

where $C_t = \{c_t^1, c_t^2, ..., c_t^k\}$ and $X = \{x^1, x^2, ..., x^k\}$.

The generalized bound for the Minkowski metric based on the L_p distortion measure can be found as follows:

$$If \qquad \sum_{i=1}^{s} |x^i - c_j^i|^p \geq \sqrt[\frac{n}{p}]{h^{\frac{n}{p}-1} D_{min}} \qquad (9)$$

$$then \qquad \sum_{i=1}^{k} |x^i - c_j^i|^n \geq D_{min} \qquad (10)$$

where $s \leq h \leq k$ and $p \leq n$.

This bound can be proved as follows :
Apply Lagrange multiplier technique to

$$minimize \qquad \sum_{i=1}^{h} a_i^n \qquad (11)$$

$$subject\ to \qquad \sum_{i=1}^{h} a_i = c, \qquad a_i \geq 0 \ \ \forall i. \qquad (12)$$

If minimum is at an interior point, then it is a stationary point of $f(a_i, \lambda) = \sum_{i=1}^{h} a_i^n - \lambda(\sum_{i=1}^{h} a_i - c)$ with respect to $a_i (1 \leq i \leq h)$ and λ.

Taking derivatives, $\frac{\partial f}{\partial a_i} = n a_i^{n-1} - \lambda = 0 \ \forall i$.
Hence $a_i = (\lambda / n)^{\frac{1}{n-1}} \ \forall i$ (which implies $a_i = a_j \ \forall i, j$) and so, to make $\frac{\partial f}{\partial \lambda} = 0$, $a_i = c/h \ \forall i$.
Here $\sum_{i=1}^{h} a_i^n = \sum_{i=1}^{h} (c/h)^n = h(c/h)^n = h^{1-n} c^n$.
The next step is to prove that the climax $\sum_{i=1}^{h} a_i^n = h^{1-n} c^n$ is the minimum point, and so to prove the following proposition.

$$If \qquad \sum_{i=1}^{h} a_i = c \qquad (13)$$

$$then \qquad \sum_{i=1}^{h} a_i^n \geq h^{1-n} c^n \qquad (14)$$

where $n \geq 1$, $h \geq 1$, and $a_i \geq 0$ for all i.

By induction, when $h = 1$, Eq. 13 reduces to $a_1 = c$ and Eq. 14 to $a_1^n \geq c^n$. It is true for $h = 1$. Assume it is true for $h - 1$. By using the Lagrange multiplier technique, if the minimum of $\sum_{i=1}^{h} a_i^n$ is at an interior point ($a_i > 0$ for all i) then this must be at the point where $a_i = c/h$ for all i, at which point $\sum_{i=1}^{h} a_i^n = h^{1-n} c^n$. At a non-interior point (without loss of generality, a_h=0),

$\sum_{i=1}^{h} a_i^n = \sum_{i=1}^{h-1} a_i^n \geq (h-1)^{1-n} c^n > h^{1-n} c^n$ (assuming $c > 0$).

The minimum cannot be at the non-interior point since the value there is greater than at the interior point already found and hence the value where $a_i = h/c$ is in fact the minimum. The proof is completed.

Hence if $c \geq \sqrt[n]{h^{n-1} D_{min}}$,
then $\sum_{i=1}^{h} a_i^n \geq h^{1-n} c^n \geq h^{1-n}(h^{n-1} D_{min}) = D_{min}$.
Set $a_i = b_i^p$, hence if $\sum_{i=1}^{h} b_i^p \geq \sqrt[n]{h^{n-1} D_{min}}$, then $\sum_{i=1}^{h} b_i^{pn} \geq D_{min}$.
Set $pn = m$, hence if $\sum_{i=1}^{h} b_i^p \geq \sqrt[\frac{m}{p}]{h^{\frac{m}{p}-1} D_{min}}$, then $\sum_{i=1}^{h} b_i^m \geq D_{min}$.
Set $b_i = |x^i - c_j^i|$, the bound for Minkowski metric based on L_p metric is derived.

If Eq. 9 is met, then C_j cannot be the nearest neighbour to X for the Minkowski metric of order n. This bound has the following properties :

(1) Set $s = p = h = 1$, it is hypercube approach.

(2) Set $p = 2$ and $n = 2$, it is the partial distortion search (PDS) for the Euclidean metric.

(3) Set $p = n$, it is PDS for L_p distortion measure.

(4) Set $n = 2$, $p = 1$ and $h = k$, it is absolute error inequality (AEI) criterion.

(5) Set $n = 2$ and $p = 1$, dubbed improved absolute error inequality (IAEI)[7] criterion by authors, it provides a tighter bound than the absolute error inequality (AEI) criterion.

(6) For the Minkowski metric of order n, this bound provides the elimination criterion from L_1 metric to L_n metric and also provides the advanced approach by adapting parameters s and h from 1 to k, i.e., this bound can be separated into several sections. For 13-dimensional coefficients and Euclidean metric, we can separate this bound into four sections. These four sections are to set h=1 to check the first dimension-difference, h=4 for the sum from the first dimension-difference to the fourth, h=9 for the sum from the first dimension-difference to the ninth and h=13 for the sum of all dimension-differences.

3 FAST CODEWORD SEARCH ALGORITHM

Here we briefly introduce the minimax method [8] at first and then combine the bound for Minkowski metric with the minimax method to get a very fast codeword searching algorithm, that is to use the step 1 to step 4 of the minimax method as the tentative match approach with IAEI and PDS to improve the searching speed. The other approach combining the minimax method with the absolute error inequality criterion is also described in this section.

3.1 Minimax Method

The minimax method is to use the codeword with the minimum value of the maximum dimension-distortion as the tentative match approach, the hypercube approach and the partial distortion search (PDS). The minimax method is depicted as follows :

step 1: For the given test vector X and codebook C, calculate the absolute error $e_{ij} = |x^i - c_j^i|$, $i = 1, 2, ..., k$, $j = 1, 2, ..., N$.

step 2: Find the maximum component of each error vector, that is to find $max_i e_{ij}$ for each codeword.

step 3: Find the minimum neighbour $l = \arg min_j max_i e_{ij}$.

step 4: Find the square Euclidean distortion $D_{min} = \sum_{i=1}^{k} e_{il}^2$.

step 5: Use the hypercube approach, i.e., if $max_i e_{ij} \geq \sqrt{D_{min}}$, then c_j will not be the nearest neighbour to X. Use the PDS to delete the rest of the codewords.

3.2 New Algorithm

The new algorithm is generated using the codeword with the minimum value of the maximum dimension-distortion as the tentative match approach, improved absolute error inequality (IAEI) criterion and partial distortion search (PDS). This new fast codeword searching algorithm is described as follows :

step 1: For the given test vector X and codebook C, calculate the absolute error $e_{ij} = |x^i - c_j^i|$, $i = 1, 2, ..., k$, $j = 1, 2, ..., N$.

step 2: Find the maximum component of each error vector, that is to find $max_i e_{ij}$ for each codeword. For convenience, interchange the maximum component of error vector with e_{1j}.

step 3: Find the minimum neighbour $l = \arg min_j max_i e_{ij}$.

step 4: Find the square Euclidean distortion $D_{min} = \sum_{i=1}^{k} e_{il}^2$.

step 5: If $\sum_{i=1}^{s} e_{ij} \geq \sqrt{h D_{min}}$, then c_j will not be the nearest neighbour to X, where $s \leq h \leq k$. Use the PDS to delete the rest of the codewords.

In this new fast codeword searching algorithm, for $s = h = 1$, it is the same as the hypercube approach in the step 5 of the minimax method. By adapting the values of s and h from 1 to k, this algorithm eliminates a very large number of multiplications.

3.3 Minimax Method with AEI Approach

In this paper, we compare the new fast codeword searching algorithm described in previous sub-section with the minimax method and the minimax method including the absolute error inequality criterion. The approach of the minimax method including AEI is described as follows :

step 1: For the given test vector X and codebook C, calculate the absolute error $e_{ij} = |x^i - c_j^i|$, $i = 1, 2, ..., k$, $j = 1, 2, ..., N$.

step 2: Find the maximum component of each error vector, that is to find $max_i e_{ij}$ for each codeword. For convenience, interchange the maximum component of error vector with e_{1j}.

step 3: Find the minimum neighbour $l = \arg min_j max_i e_{ij}$.

step 4: Find the square Euclidean distortion $D_{min} = \sum_{i=1}^{k} e_{il}^2$.

step 5: Use the hypercube approach, i.e., if $max_i e_{ij} \geq \sqrt{D_{min}}$, then delete the codeword c_j. Use AEI criterion, i.e., if $\sum_{i=1}^{s} |x^i - c_j^i| \geq \sqrt{k D_{min}}$, then c_j will not be the nearest neighbour to X, where $s \leq k$. Use the PDS to delete the rest of the codewords. Here the AEI criterion is applied by adapting s from 1 to k.

4 EXPERIMENTAL RESULTS

The test materials for these experiments consist of two hundred words recorded from one male speaker. The speech is sampled at a rate of 16 kHz and 13-dimensional cepstrum coefficients with variance weighting are computed over 20 ms-wide frames with a 5 ms frame shift. A total of 20,030 analyzed frames are used in the codeword searching experiments. 64, 256 and 1024 codewords with Euclidean distortion measure are used in these experiments.

Experiments are carried out to test the performance of the minimax method, the minimax method with absolute error inequality elimination rule and the new fast searching algorithm described above. n=2 and p=1 are chosen for this new bound. The bounds for Minkowski metric are separated into four sections. These four sections are to set h=1 to check the first dimension-difference, h=4 for the sum from the first dimension-difference to the fourth, h=9 for the sum from the first dimension-difference to the ninth and h=13 for the sum of all dimension-differences.

The experimental results are depicted in Table 1, 2 and 3. Table 3 shows that this new fast codeword searching algorithm improves more than 77% and 21% multiplication operations compared with minimax method and minimax method with AEI criterion for 1024 codewords.

method	mul.	cmp.	add.
Minimax	2,865	20,035	19,053
Minimax_AEI	1,400	23,581	22,600
NEW	1,256	22,262	21,353

Table 1: computational complexity of codeword search for 64 codewords ($\times 10^3$)

method	mul.	cmp.	add.
Minimax	5,088	75,640	70,813
Minimax_AEI	1,901	83,135	78,308
NEW	1,584	79,700	74,949

Table 2: computational complexity of codeword search for 256 codewords ($\times 10^3$)

method	mul.	cmp.	add.
Minimax	7,569	292,892	272,682
Minimax_AEI	2,133	305,783	285,573
NEW	1,671	299,002	278,865

Table 3: computational complexity of codeword search for 1024 codewords ($\times 10^3$)

5 CONCLUSIONS

In this paper, the bound for Minkowski metric is derived. For the Euclidean distortion measure, $p = 1$ and $n = 2$, this bound is tighter than the standard absolute error inequality (AEI) elimination rule (k instead of h). This bound provides a better criterion than the standard AEI. For $n = 2$, $p = 1$, $h = 1$ and $s = 1$, this bound is the same as the hypercube approach for L_2 metric. For $p = n = 2$, it is the same as the partial distortion search (PDS) for the Euclidean metric. For the Minkowski metric of order n, this bound provides the elimination criterion from L_1 metric to L_n metric and also provides the advanced approach by adapting parameters s and h from 1 to k. This novel bound of the Minkowski metric extends the AEI from the Euclidean metric to higher order distortion measures and also provides better criterion. Experimental results confirm this new bound. This bound can also be applied to codebook design and speech recognition.

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