Electromagnetic articulograph (EMA) systems are useful to study the motor control of speech articulators and also to construct models of the speech production process. In the EMA system, the position of the receiver coil is predicted on the basis of a field function representing a spatial pattern of the magnetic field in relation to the relative position between the transmitter and receiver coils. This paper presents a new method of representing the magnetic field by using a multivariate spline function to overcome the problem of the field pattern having local fluctuations caused by interference between the transmitting signals. A procedure for determining the receiver position is also presented, and the piecewise property of the basis functions enables the spline function to flexibly approximate the field pattern and to attain a high measurement accuracy: the mean error in estimating the receiver position was less than 0.1 mm for a 14 × 14-cm measurement area.

1. Introduction

Electromagnetic articulograph (EMA) systems [1,2] are useful in the research of speech production because they can measure the activity of the articulatory organs to a high degree of spatial and temporal resolution. These systems are also effective for constructing a dynamic articulatory model [3], and for studying articulatory-based speech synthesis [4] and acoustic-to-articulatory inversion problems [5].

The EMA system uses three transmitter coils driven by alternating currents of different frequencies, and measures the signal strength electro-magnetically induced in the receiver coil. For determining the position of the receiver coil, the spatial pattern of the magnetic field is represented as a voltage-to-distance function (VDF) in relation to the relative position between the transmitter and receiver coils. Kaburagi and Honda suggested in their study [6] that the magnetic fields produced were globally smooth, but had local fluctuations because of the interference between transmitter signals. To overcome this problem, they proposed an adaptive calibration method in which the parameters included in the VDF were locally calibrated adaptive to the receiver position, and it resulted in a higher measurement accuracy.

This paper presents a new method for representing the magnetic field and describes how the receiver position can be calculated using the proposed field function. The field pattern in our method is expressed using the multivariate B-spline function [7], which can depict a smooth curved surface as a linear combination of piecewise basis functions. The position of the receiver coil is then determined so that the difference between the measured and predicted signal strengths is minimized. The spline function generally has a superior capability to represent a curved surface, even when it has a local fluctuation or discontinuity, because each basis function is defined locally and there are freedoms in the selection of the rank and the number of the basis functions. These flexibilities allow the spline function to accurately represent the magnetic field, although it suffers from the locality problem described earlier. In addition, the spline function can provide a closed form representation of an overall field pattern. Therefore, the proposed method circumvents using the computationally inefficient pattern matching in [6] required to discover the local calibration data.

This paper first explains the spline-based representation of the magnetic field, and presents the method for estimating the receiver position. Then, we present experimental results conducted to examine the accuracy of the proposed method.

2. Magnetic field representation

The EMA system used in this study (Carstens Articulograph AG100, Germany) has three transmitter coils positioned at vertices \( T_1, T_2, \) and \( T_3 \) of a regular triangle with side lengths of 64.18 cm (Fig. 1). The measurement area is set inside the triangle \( T_1T_2T_3 \) as a 14 × 14 cm rectangle so that it covers the entire range of measurement points on the articulators.

The strength of the magnetic field \( v(x, y) \) at a point in the EMA coordinate system is represented using a multivariate B-spline function [7]:

\[
\log\{v(x, y)\} = \sum_{p=1}^{P} \sum_{q=1}^{Q} c_{pq} N_p(x) N_q(y) \quad (1)
\]

where \( N_p(x) \) and \( N_q(y) \) respectively represent piecewise polynomial functions of rank \( m \), each of which becomes...
zero outside a certain interval along the axis. $c_{pq}$ denotes the weight in summing combinations of the basis functions.

Because of the piecewise property of the basis function, the spline function generally provides a good result when interpolating data and approximating the overall shape of a curved surface. In addition, the smoothness of the surface can be controlled by parameter $m$ which determines the polynomial order of the basis function.

### 2.1. Construction of the basis functions

The basis functions $N_p(x)$ and $N_q(y)$ are uniquely determined by setting the number and the positions of the internal nodes, irrespective of the data samples to be interpolated. Internal nodes of the two-variate spline function in Eq. (1) can be arranged in a rectangular region $R = [a, b] \times [c, d]$ as follows:

$$a = \xi_{1-m} = \cdots = \xi_{-1}$$
$$= \xi_0 < \xi_1 < \cdots < \xi_p < \cdots < \xi_{P-m+1}$$
$$= \xi_{P-m+2} = \cdots = \xi_{P-1} = \xi_P = b$$

$$c = \zeta_{1-m} = \cdots = \zeta_{-1}$$
$$= \zeta_0 < \zeta_1 < \cdots < \zeta_q < \cdots < \zeta_{Q-m+1}$$
$$= \zeta_{Q-m+2} = \cdots = \zeta_{Q-1} = \zeta_Q = d$$

where $\xi_p$ and $\zeta_q$ represent nodes along the $x$ and $y$ axes, respectively. Note that $m$ nodes overlap on both sides, and that the number of non-overlapping nodes ($n$) equals $P - m$ and $Q - m$. Each basis function is uniquely determined from the positions of successive $m + 1$ nodes, and has the following piecewise property:

$$N_p(x) \begin{cases} > 0, & \xi_{P-m} < x < \xi_P \\ = 0, & \text{otherwise} \end{cases}$$

### 2.2. Determining the weighting coefficients

For given data samples, the spline function can be designed so that it forms a curved surface that interpolates them. In our method, these data are given as the calibration data samples at the points where lines drawn at equal intervals in the horizontal and vertical directions of the entire measurement area intersect, as shown in Fig. 1. For the spatial alignment of the internal nodes, the rectangular region $R = [a, b] \times [c, d]$ is taken to coincide with the measurement area, and the node position is selected just like the sampling points of the calibration data.

When the measured field strength sampled at position $(x_i, y_j)$ is $v_{ij}$, the weighting coefficients $c_{pq}$ in Eq. (1) can be determined so that the spline function simultaneously satisfies the following equations [7]:

$$\log v_{ij} = \sum_{p=1}^{P} \sum_{q=1}^{Q} c_{pq} N_p(x_i) N_q(y_j)$$

where $1 \leq i \leq N$ and $1 \leq j \leq N$. $N$ is the number of calibration data samples of each axis. $N = m + n$ must be satisfied for the number of the nodes ($n$) and the rank of the basis functions ($m$).

### 3. Determining the receiver position

This section describes the method for measuring the position of the receiver coil. This problem can be regarded as the determination of three unknown variables, i.e., position variables $x$ and $y$ and the tilt angle $\theta$, from the strength of the induced signal in the receiver coil. Based on the field representation [Eq. (1)], the induced signal strength can be written as a function of the unknown variables as

$$\hat{e}_l(x, y, \theta) = v(x, y) \cos \theta$$

where the logarithm of the field strength $v(x, y)$ is represented by the spline function, and $l$ is an index of the transmitter coils ($l = 1, 2, 3$). Next, we introduce a criterion of the difference between received and predicted signals:

$$S(x, y, \theta) = \sum_{l=1}^{3} \left\{ \log e_l - \log \hat{e}_l \right\}^2$$

where $e_l$ denotes the received signal strength. The position of the receiver coil is determined, as well as the rotation angle, for the measured signal strength so that the error criterion is minimized. This optimization problem becomes nonlinear since the magnetic field is represented by the piecewise polynomial functions. Therefore, we employ an iterative procedure based on the Gauss-Newton method.

Let vectors $\mathbf{e}$ and $\hat{\mathbf{e}}$ respectively represent measured and predicted logarithmic signals as

$$\mathbf{e} = (\log e_1, \log e_2, \log e_3)^{t}$$

$$\hat{\mathbf{e}} = (\log \hat{e}_1, \log \hat{e}_2, \log \hat{e}_3)^{t}.$$
\( x \) represents the unknown parameters whose values are incrementally modified by a vector \( \Delta x \) in the iterative procedure:

\[
x = (x, y, \theta)^T
\]

(11)

\( \Delta x = (\Delta x, \Delta y, \Delta \theta)^T \).

(12)

\( \Delta x \) can be determined by solving a normal equation

\[
A^T A \Delta x = A^T (e - \hat{e})
\]

(13)

where \( A \) is the following Jacobian matrix

\[
A = \begin{pmatrix}
\partial \log \hat{e}_1 / \partial x & \partial \log \hat{e}_1 / \partial y & \partial \log \hat{e}_1 / \partial \theta \\
\partial \log \hat{e}_2 / \partial x & \partial \log \hat{e}_2 / \partial y & \partial \log \hat{e}_2 / \partial \theta \\
\partial \log \hat{e}_3 / \partial x & \partial \log \hat{e}_3 / \partial y & \partial \log \hat{e}_3 / \partial \theta 
\end{pmatrix}
\]

(14)

representing the sensitivity of the unknown parameters over the predicted signals.

The procedure for determining the optimal values of the unknown parameters is summarized as follows:

1. Given the measured signal strength \( e \) and the initial values of the unknown parameters \( x^0 \), set the counter \( k \) at zero.

2. Calculate the predicted signals \( \hat{e} \) and the value of each component of the Jacobian matrix \( A \) for \( x^k \). Then solve the normal equation [Eq. (13)].

3. Update the values of the unknown parameters as

\[
x^{k+1} = x^k + \alpha \Delta x^k
\]

(15)

where \( \alpha \) is a reduction parameter (\( 0 < \alpha \leq 1 \)).

4. When a convergence is obtained, quit the procedure. Otherwise set the counter as \( k = k + 1 \) and repeat from the second step.

When computing components of the Jacobian matrix, the values of the basis functions and their partial derivatives, \( \partial N_n(x) / \partial x \) and \( \partial N_n(y) / \partial y \), should be evaluated for a specific value of \( x^k \). These calculations can be effectively performed based on de Boor’s incremental algorithm [8]. In our study, the reduction parameter in the third step is expressed as \( \alpha = 0.1 L \), where \( L \) is an integer ranging from one to ten, and \( L \) is selected so that the prediction error of the received signal strength [Eq. (8)] is minimized.

4. Experiment

4.1. Accuracy for representing the magnetic field

An experiment was conducted to examine the accuracy of the spline function for representing the strength of the magnetic field. Calibration and test data samples were gathered, as shown in Fig. 1, at each lattice point in the measurement area. Eight sets of calibration data were measured while changing sample number \( N \) from three to ten. The sample number of the test data was selected at \( N = 15 \) so that the interval between the adjacent sampling points was 1 cm.

Figure 2 shows the error in representing the magnetic field as a function of the number of the calibration data samples, when the rank of the basis function was set at three, four, and five. The ordinate represents a mean of three trials, where errors for 15 × 15 test samples were also averaged in each trial. To obtain these results, the number of internal nodes was set at \( n = N - m \) for the given number of the calibration data \( N \) and the rank of the basis function \( m \), and the weighting coefficients of the spline function were determined. Then, the position of each test sample \( (x_i, y_j) \) was substituted into the spline function, and the relative error between the predicted \( v(x_i, y_j) \) and measured \( v_{ij} \) signal strengths was evaluated as

\[
100 \times |v(x_i, y_j) - v_{ij}| / v_{ij} (\%)
\]

The figure indicates that the error was less than 0.06% when the number of calibration samples was greater than three. The influence of rank \( m \) was relatively small, but an increase of the error was observed for \( m = 5 \) when the sample number was greater than seven. The results suggest that the logarithm of the field is relatively flat and well represented by the polynomial basis functions of the second order.
4.2. Accuracy in determining the receiver position

Figure 3 shows the experimental result conducted to examine the measurement accuracy of the receiver position. In this experiment, the magnetic field representation was obtained using the same set of calibration data as the previous one, the receiver position \((x, y)\) was calculated from the measured signal strength of each test sample, and the result was compared with the actual position \((x_i, y_j)\) as \(\sqrt{(x-x_i)^2 + (y-y_j)^2}\). The measurement error was less than 0.1 mm for \(N > 4\), which was smaller than the one obtained in the previous study [6]. The tendency of the errors in Figs. 2 and 3 are similar, indicating that the accurate representation of the magnetic field would result in a fine measurement accuracy.

The measurement error was plotted in Fig. 4 versus the iteration number of the Gauss-Newton method. It is clear from the figure that the iterative procedure has a rapid convergence: three or four iterations are enough to reach a saturation level in the decrease of the error.

Figure 5 shows the influence of the tilt and off-center misalignment of the receiver coil. The measurement error was obtained for several combinations of lateral displacements and rotation angles with respect to the tilt and twist. The results indicate that it is very important to place the receiver coil exactly in the midsagittal plane [2] because the error due to the rotation is more significant when the lateral displacement is increased. When the rotation angles are 20 degrees for both tilt and twist, the lateral displacement must be less than 2 mm to keep the error less than 1 mm.

5. Conclusion

A spline-based method was proposed to represent the magnetic field and a procedure was also presented to determine the receiver position. Experimental results showed that a fine measurement accuracy was achieved when the receiver was in the midsagittal plane, but the influence of the off-center misalignment and rotation was very large. We will extend the proposed method to a three-dimensional EMA system [9] and remove these problems.

6. References