An Efficient Viterbi Algorithm on DBNs

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Abstract

DBNs (Dynamic Bayesian Networks) [1] are powerful tool in modeling time-series data, and have been used in speech recognition recently [2,3,4]. The “decoding” task in speech recognition means to find the viterbi path [5] (in graphical model community, “viterbi path” has the same meaning as MPE “Most Probable Explanation”) for a given acoustic observations. In this paper we describe a new algorithm utilizes a new data structure “backpointer”, which is produced in the “marginalization” procedure in probability inference. With these backpointers, the viterbi path can be found in a simple backtracking. We first introduce the concept of backpointer and backtracking; then give the algorithm to compute the viterbi path for DBNs based on backpointer and backtracking. We prove that the new algorithm is correct, faster and more memory saving comparison with old algorithm. Several experiments are conducted to demonstrate the effectiveness of the algorithm on several well known DBNs. We also test the algorithm on a real world DBN model that can recognize continuous digit numbers.

1. Introduction

Based on junction tree inference algorithm [6,7,8], A.P. David [6,7] gave the first algorithm to get the MPE in static BNs. We call it as “DAP algorithm”. It first performs a “collect evidence” procedure (forward pass) using a “MAX flow”[6,7]; then find the maximal entry in the root clique’s potential and set it to 1, set other entries to 0s; In the backward pass, DAP algorithm “distribute evidence” based on the changed root potential and in the same time get the MPE. K. Murphy extended the idea to DBNs based on his “1.5 slice junction tree algorithm” [1,9]. We call it as KPM algorithm and will introduce in later section.

KPM algorithm suffers from two problems. First it need a full “backward pass” procedure, this is slow, and has computational complexity not only depend on the structure of the junction tree, but also the size (number of entry) of each cliques’ potentials in junction tree. Furthermore it requires all cliques and separators potentials in the “backward pass”. All the potentials produced in the “forward pass” need to be saved. This will cause great overhead not only on memory, but also performance of the algorithm. We modify KPM algorithm by introducing data structure “backpointer” and function “backtracking”. See section 3 for algorithm description and experiment results in section 4.

In this paper we first introduce concepts of “backpointer” and “backtracking”, see section 2. Section 3 presents the KPM algorithm, the new algorithm. In section 4 we list some experiments results on both algorithms, compare the performance and memory consummation on several well known DBNs, and on a real world two stream speech DBN models. Section 5 is the conclusion.

2. Backpointer and Backtracking

2.1. Backpointer

The idea of “backpointer” has long been used in speech recognition, where it is used to do backtracking in decoding of HMMs to get the viterbi path [5]. Here we define a “backpointer” as an integer array produced by a “max marginalization” procedure (a “max marginalization” means to pass a “max flow” when perform a marginalization), each element in a backpointer is the index of a configuration (entry) in the potential that be marginalized.

For example, potential $P_1$ contain 3 binary variables $X_0$, $X_2$, and $X_1$ (We say that $P_1$ has “domain” $X_0$, $X_2$ and $X_1$), and has joint probability distribution $p(X_0, X_2, X_1) \sim [.15, .20, .05, .12, .27, .03, .11, .07]$. While a marginalization of $P_1$ to produce $P_2$ that is a potential on variables $X_1$ and $X_2$ ($P_2$ has “domain” $X_1$ and $X_2$), i.e. we marginalize probability distribution $p(X_0, X_2, X_1)$ to probability distribution $p(X_0, X_1)$. We get the potential $P_2$ with distribution is [.15, .20, .27, .07]. At the same time, we also get the corresponding backpointer which is [1, 2, 5, 8]. The first element “1” in the backpointer means that the first element in $P_2$ ← .15” is passed from the 1st element of $P_1$. Similarly, the third element “5” in the backpointer means that the third element of $P_2$ is passed from the 5th element — .27” from $P_1$. A backpointer has the same size and shape as the produced result potential in a max marginalization operation except that they have different data type, i.e. integer instead of floating type.

2.2. Backtracking

A backtracking procedure that utilizes a backpointer between a pair of cliques is illustrated in Fig.1. Where clique $A$ marginalizes to separator $S$ (not shown in Fig.1), produce a backpointer $BP$; then multiply $S$ to clique $B$. Now we want to find the entry in $A$ that corresponds to an entry “1” in $B$. First we find the entry in $BP$ that corresponds to “1” in $B$. For a given entry in $B$, there is one and just one corresponding entry in $S$ (see the definition of potential multiplication in a junction tree), which is determined by
domain of $B$ and $S$. In this case, the entry in $S$ that
corresponding to entry “$i$” in $B$ is “$j$”, as illustrated in Fig.1.
From $BP$ we get $k ← BP_j$, meaning that the corresponding
entry in $A$ is “$k$” for entry “$i$” in $B$. Fig.2a present the algorithm
for backtracking between a pair of neighbor cliques.

**BACKTRACK_IN_PAIR(Parent, Child, BP, idx)**
1. $i ← idx$;
2. $j ← CLIQUE_TO_BACKPOINTER(Parent, BP, i)$;
3. $k ← BP_j$;
4. return $k$

Fig. 2a: backtracking in a pair of cliques

**BACKTRACK_IN_TREE(TREE, BP_TREE, idx)**
1. RESULT_TREE.ROOT ← idx;
2. for Parent ← TREE.PARENT
3. for Child ← TREE.CHILDREN[Parent]
4. $BP ← BP_TREE(Parent, Child)$;
5. idx ← RESULT_Parent;
6. RESULT_Cands ← BACKTRACK_IN_PAIR(Parent, Child, BP, idx)
7. return RESULT

In Fig.2a “Parent” and “Child” are two cliques; “BP” is the
backpointer between the “Parent” and “Child”; “idx” is the
index of an entry in “Parent” that need be backtracked.
Function “CLIQUE_TO_BACKPOINTER” just computes entry “$j$” in $BP$
that correspond to entry “$i$” in “Parent”. As
illustrated in Fig.1. Backtracking can easily be extended to
backtrack through a chain of cliques, or to backtrack in a
tree. Backtracking in a tree means given an entry of the root
clique, perform a backtracking level by level, eventually
reach each leaf cliques, and record each corresponding entries
in each cliques. Fig.2b and Fig.2c illustrate to
perform a backtracking in a tree. The $BP_TREE$ is the
backpointers of the tree, which is a 2D array.

All computations in the backtrackings are integer type and
independent on the size of each clique’s potentials. This is
different with potential’s multiplication, division and
marginalization which is floating point computation and has
complexity known as the direct ratio to size of each
potentials in operation. Briefly backtracking in a pair of
potentials has constant complexity $O(1)$, but the backward
pass in a pair of potentials has complexity $O(|A| + |B|)$,
where $|A|$, $|B|$ are sizes of $A$ and $B$ respectively. And in the
same time, backtracking need store $|S|$ integer data, but
backward pass need store $(|A| + |B| + |S|)$ floating data.

2.3. Proof of Backtracking

The DAP algorithm has been proved in [6]. The KPM
algorithm is also correct [1]. We can prove that the
backtracking based on backpointer can produce the same
result as the DAP algorithm.

Consider a simple junction tree that has only two cliques $A$
and $B$, where $A$ is the leaf and $B$ is the root. $S$ is the
separator. (Fig.1). In the forward pass, we get the
backpointer $BP$. $BP_i ← k$ means that the $k$th element in $A$
passed its value to the $j$th element in $S$. The arrow from the $i$th
element of $B$ to the $j$th element of $S$ is not backpointer, but a
deterministic arrow. There is one and only one element in $S$
corresponding to a specific element in $B$ (for more detail, see
the description of junction tree algorithm). Now assume the
$i$th element is the maximal element in $B$.

From the algorithm in Fig.2a we know that the configuration
of the $k$th element in $A$ will become a part of the MPE. We
now prove that the $k$th element is still the element we need to
find when performing the DAP algorithm. Denote $POTA_X$ as
the potential of a clique or a separator $X$, and $POTA_k$ as
the $k$th entry of $POTA_X$. Now suppose the $i$th element in $B$ is the
maximal, and set $POTA_i$ to 1, set all other entry to 0s. When performing “distribute evidence”, the DAP algorithm do:

1. $POTA_i ← POTA_i / POTA_k$
2. $POTA_k ← MARGINALIZE(POTA_k)$
3. $POTA_k ← POTA_k + POTA_i$

We know $BP_i ← k$, means that $POTA_k$ is the maximal
element in $POTA_k$ (const), where $CONS(j)$ indicate a set of
elements that “consistent with” $j$ (consistent means that two elements
have the same configuration in same domains) and
$k ∈ CONS(j)$. After step 1, the $POTA_k$ is still the maximal
element in $POTA_k$ (const), and has the value 1, because all
elements in $POTA_k$ (const) need to divide by same $POTA_k$. After
step 2, $POTA_k$ equals to 1 (passed from the $i$th element in $B$).

Then after step 3, the element $POTA_k$ is still the maximal
element in $POTA_k$ (const) because all elements in $POTA_k$ (const)
will multiply a const value 1, and we have already known
that the original $POTA_k$ is the maximal in $POTA_k$ (const)
after step 1. Thus we know that the DAP algorithm and the
backpointer based backtracking algorithm can produce the
same MPE result. This procedure can similarly be
generalized to junction tree with more than two cliques or a
junction trees chain produced from a DBN.

3. Compute MPE on DBNs

3.1. The KPM Algorithm

KPM algorithm produces an independent junction tree for
each time slice; all these junction trees have the same
structure except the first. Then glue these junction trees by
the “forward interface”, see Fig.3. A “forward interface”
means variables in a single time slice in the DBN that has an
arrow pointer to later time slice variables. In Fig.3, $J_t$ to $J_{t+1}$
are the $T$ junction trees induced from the original DBN, $I_t$ to
$I_{t+1}$ are the $T-1$ interface cliques for corresponding junction
trees. Clique $D$ is the input clique in $J_t$, $C_t$ is the root and
output clique of $J_t$. See Fig.4 for KPM algorithm.

**KPM_ALGORITHM(DBN, EVIDENCE)**
1. $J_TREE[1:T] ← CONSTRUCT_JTREES_FROM_DBN(DBN)$;
2. $\{CP, SP\} ←$
NEW_ALGORITHM(\text{DBN, EVIDENCE})
1 \text{JTREES}_t \leftarrow \text{CONSTRUCT_JTREES_FROM_DBN(DBN)};
2 (CP_t, SP_t) \leftarrow \text{INITIALIZE_JTREE(JTREES}_t, \text{DBN, EVIDENCE})
3 (CP_t, SP_t, BP_t) \leftarrow \text{COLLECT_EVIDENCE_BP(JTREES}_t, \text{CP}_t, \text{SP}_t);
4 delete CP_t, SP_t, BP_t;
5 for t \leftarrow 2 : T
6 (CP_t, SP_t) \leftarrow \text{INITIALIZE_JTREE(JTREES}_t, \text{DBN, EVIDENCE})
7 CP_t^0 \leftarrow CP_t^0 * IP_t^0;
8 (CP_t, SP_t) \leftarrow \text{COLLECT_EVIDENCE(JTREES}_s, \text{CP}_s, \text{SP}_s);
9 IP_t \leftarrow \text{MARGINALIZE}(CP_t^0);
10 (CP_t^f, MPE_t^f) \leftarrow \text{SET_MAX_CONFIG}(CP_t^f);
11 for t \leftarrow T : 1
12 CP_t \leftarrow \text{DISTRIBUT_EVIDENCE(JTREES}_s, \text{CP}_s, \text{SP}_s);
13 MPE_t \leftarrow \text{FIND_MAX_CONFIG}(CP_t);
14 CP_{t+1} \leftarrow CP_{t+1} / IP_{t+1};
15 IP_{t+1} \leftarrow \text{MARGINALIZE}(CP_{t+1})
16 CP_{t+1} \leftarrow CP_{t+1} * IP_{t+1};
17 \text{VITERBIPATH} \leftarrow \text{CONFIG_TO_INDEX(JTREES}_s, \text{MPE}_s)
18 return \text{VITERBIPATH}

Fig.5 New algorithm to find MPE in DBN

Where \text{BP}_s, \text{BP}_t are the backpointers for \text{JTREES}_s and interface clique \text{I}_t, respectively. The function “\text{COLLECT_EVIDENCE_BP}” is the same as the function “\text{COLLECT_EVIDENCE}” in Fig.4, except it produces the additional “backpointer” in each marginalization.

The function “\text{BACKTRACK_IN_TREE}” perform a backtracking in a single junction tree which is described in Fig.2b. Function “\text{BACKTRACK_IN_TREE}” is just a simple backtracking from clique \text{D}_t to clique \text{C}_{t+1} through the interface \text{I}_{t+1}, using the backpointer \text{BP}_t.

We can see that the backtracking (line 13 – 17) need not use any potentials values. All the computations are the simple integer operations and have constant complexity. Memory used to store the backpointers will be much less than to store all the potentials. We generalize the Viterbi backtracking algorithm to junction trees. The basic idea is that instead of keeping a single integer to represent the backpointer, one keeps a vector of integers, which encode the “argmax” computed in the forwards pass.

For example, consider passing a message from potential \text{P}_t(X_1, X_2, X_3) to \text{P}_2(X_1, X_2) in the forwards pass (i.e. marginalization of \text{P}_1 to \text{P}_2). The message, which will be stored on the separator, can be computed as

\text{MESSAGE}(X_1, X_2) \leftarrow \text{ARGMAX}_X_{1,2}(\text{P}_1(X_1, X_2, X_3))

We also store the backpointer, \text{BP}(X_1, X_2) \leftarrow \text{ARGMAX}_X_{1,2}(\text{P}_1(X_1, X_2, X_3))

On the backward pass, suppose (by induction) that \((X_1^*, X_2^*)\) is the optimal assignment to the separator. Then we can compute the optimal assignment to \text{P}_2 by setting
$X_1^* \leftarrow BP(X_1^*, X_3^*)$

This algorithm requires keeping backpointers for every separator and interface in each time slice. However, old clique potentials do not need to be stored, just as old alpha variables are not needed in Viterbi for HMMs [5].

4. Experiments Results

Experiments are conducted on a PC with a Pentium III XEON 933 MHz and 512M memory. Algorithms mentioned above are implemented based on BNT[9].

4.1. Experiments on 7 Typical DBNs

Experiments are conducted on 7 typical sample DBN models (all are available in BNT, “chmm(m,n)” means that the coupled HMM has “m” hidden chains and each hidden variable has “n” states). All have 10 time slices. We compare the total execution time and the maximal memory used when running both algorithms on each DBNs. See Table 1.

<table>
<thead>
<tr>
<th></th>
<th>Execute time(s)</th>
<th>Memory used(KB)</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>new KPM</td>
<td>new KPM</td>
</tr>
<tr>
<td>mildew</td>
<td>0.234</td>
<td>171</td>
</tr>
<tr>
<td>water</td>
<td>0.237</td>
<td>247</td>
</tr>
<tr>
<td>chmm(9,2)</td>
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<td>423</td>
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<tr>
<td>chmm(4,5)</td>
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<tr>
<td>BAT</td>
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<td>1,282</td>
</tr>
<tr>
<td>chmm(10,2)</td>
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<td>516</td>
</tr>
<tr>
<td>chmm(5,5)</td>
<td>5.125</td>
<td>552</td>
</tr>
</tbody>
</table>

Table 1. KPM algorithm and the new algorithm on 7 typical DBNs

4.2. A Real DBN Application on Speech Recognition

DBNs have been used in speech recognition in recent years [1,2,3]. In this experiment, we construct a DBN to recognize continuous digit strings (AURORA data). Other than normal implementations, we use 2- asynchronous observation streams to model the asynchronous property of extracted features. For simplicity, we use single Gaussian to represent the acoustic observations, and use a whole word model to represent each digit number ‘0’-’9’ and the ‘silence’. Each digit number has 16 states, and ‘silence’ has three states.

From Table 1 we see that our new algorithm can get 10%--50% faster than KPM algorithm, and can save memory about 80%--90% in the same time. Table 2 show us that the new algorithm can get higher speedup factor (>2) compare to table 1, because the model is so large, need so many memory to store all the potentials, and so many memory access makes the KPM algorithm’s performance degrade greatly than it should be. The situation of KPM algorithm becomes worse with the increasing of time slice of a DBN.

5. Conclusion

We proposed a new algorithm that can compute the viterbi path for DBNs, based on a new introduced data structure we called a “backpointer”. We prove that the new algorithm is correct. Experiments show that the new algorithm will get 10%--40% faster and save 80%--90% memory than old algorithm. The new algorithm can get even faster (>2X) than old algorithm when memory issue become much critical.

Reference