Using Linear Prediction in Spectral Domain to Decompose Speech into Modulated Components

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Abstract

The paper presents the decomposition of the speech signal into two modulated components, namely the envelope and the instantaneous frequency (IF). For this purpose, a bandpass signal is first represented as a sum of sinusoidal signals and then, by using the roots of the polynomial with complex coefficients of the signal, it is transformed in a product of elementary signals. This representation allows an easier computation of the envelope and of the IF. In order to eliminate the computation of the roots, a LPSD (linear prediction in spectral domain) algorithm is used. Thus, the two components of the signal can be computed. First, the method is applied to a synthesized signal, and then to a speech signal.

1. Introduction

The envelope and the IF are instantaneous attributes of a signal. Both depend on time. If \( s(t) \) is a real-valued signal, computation of the envelope and IF needs the analytic signal \( s_a(t) \) formed from the real-valued signal, \( s_a(t) = s(t) + j\hat{s}(t), \)
\[ \hat{s}(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{s(\tau)}{t-\tau} \, d\tau. \]

Thus, the envelope of \( s(t) \) is
\[ a(t) = \|s_a(t)\|, \]
and its IF is
\[ \phi(t) = \frac{1}{2\pi} \frac{d\theta(t)}{dt}, \]
where \( \theta(t) \) is the angle of \( s_a(t) \).

The analytic signal preserves all the information contained in the real-valued signal, but its spectrum is zero for negative frequencies.

Generally, the analytic signal is not computed using the Hilbert transform, because this method leads to a number of difficulties [4]. A practical solution for obtaining a discrete-time analytic signal is based on computing a discrete-time Fourier transform (DTFT), followed by an inverse DTFT [5]. Thus, if \( s[n], \) \( n=0,\ldots,N-1 \), is obtained by sampling a bandlimited real-valued signal, \( s[n]=s(nT_s) \) where \( T_s \) is the sample time, the DTFT is
\[ X(f) = T_s \sum_{n=0}^{N-1} s[n] e^{-j2\pi nfT_s}. \]

Then, the \( N \)-point DTFT of the analytic signal is obtained by replacing with zeros the DTFT coefficients that correspond to negative frequencies:
\[ X_a[k] = \begin{cases} X[0], & \text{for } k = 0 \\ 2X[k], & \text{for } 1 \leq k \leq \frac{N}{2} - 1 \\ X[N/2], & \text{for } k = \frac{N}{2} \\ 0, & \text{for } \frac{N}{2} + 1 \leq k \leq N - 1. \end{cases} \]

Finally, the analytic signal is obtained using an \( N \)-point inverse DTFT,
\[ s_a[n] = \frac{1}{N T_s} \sum_{k=0}^{N-1} X_a[k] e^{j2\pi kn/N}. \]

The imaginary part of an analytic signal is a version of its real part (i.e. the real signal) with a 90° phase lag. Thus, \( \cos(x) \) are transformed in \( \sin(x) \), and \( \sin(x) \) are transformed in \( -\cos(x) \).

One of the most important signals used in simulations is a complex signal denoted as \( s_1(t) \),
\[ s_1(t) = \sum_{k=1}^{M} b_k e^{j k \Omega}, \]
where \( \Omega = 2\pi n T_s \) is the fundamental angular frequency of \( s_1(t) \) and \( b_k \) are the complex coefficients (each of them contains the amplitude and the phase of the \( k \)-th harmonic),
\[ b_k = |b_k| e^{j \phi_k}. \]

The real and imaginary part of the signal \( s_1(t) \) are
\[ \text{real}(s_1(t)) = \sum_{k=1}^{M} |b_k| \cos(k \Omega + \phi_k), \]
\[ \text{imag}(s_1(t)) = \sum_{k=1}^{M} |b_k| \sin(k \Omega + \phi_k). \]
It can be seen that the imaginary part of \( s_1(t) \) is the Hilbert transform of the real part and thus \( s_1(t) \) is an analytic signal.

Let \( s_1(t) \) a real signal,

\[
s_2(t) = \sum_{k=-N}^{N} b_k e^{j\Omega k}, \quad h_0 = b_0 = 0,
\]

where * denotes complex-conjugate.

This signal can also be expressed as

\[
s_2(t) = s_1(t) + s_1^*(t) = 2\text{real}(s_1(t)),
\]

or,

\[
\text{real}(s_1(t)) = \frac{1}{2} s_2(t).
\]

It follows that \( s_1(t) \) is

\[
s_1(t) = \frac{1}{2} s_2(t) + j \frac{1}{2} \tilde{s}_2(t),
\]

that is, \( s_1(t) \) is the analytic signal of \((1/2)s_2(t)\).

Equations (13), (14) and (15) are not true if in equation (8) there is a \( b_0 \neq 0 \).

By using equation (4) for computing the IF, where \( s_1(t) \) must be replaced by \( s_1(t) \), the IF can also take negative values, which are not physically justifiable. Therefore, in [1] and [2] an algorithm is offered that decomposes the signal \( s_1(t) \) in two components: one whose envelope is identical with the envelope of \( s_1(t) \) and another whose envelope is time independent (is constant), and whose IF is always positive. This decomposition, to be presented in section 3, is made by using the LPSD algorithm.

The paper is organized as follows. Section 2 presents the LPSD algorithm, by a comparison with its classical form, namely linear prediction in time domain. In section 3 the LPSD algorithm is used to decompose a bandpass signal into its envelope and positive IF. Section 4 contains the results of simulation using a synthesized bandpass signal and a real speech signal.

2. Linear prediction in time and spectral domains

2.1. In time domain

Let \( x(n) \) a sequence with complex valued samples, denoted as \( x(0), x(1), \ldots, x(N) \). The estimation value of \( x(k), k=0,1,\ldots,N \), denoted by \( \hat{x}(k) \) is

\[
\hat{x}(k) = -\alpha_1 x(k-1) - \alpha_2 x(k-2) - \ldots - \alpha_L x(k-L)
\]

where \( \alpha_i, i=1,\ldots,L \) are the prediction coefficients and \( L \) is the prediction order. Let \( ep(k) \) the prediction error of the sample \( x(k) \)

\[
ep(k) = x(k) - \hat{x}(k).
\]

Then, the prediction coefficients are determined by minimizing the energy of prediction error,

\[
Ep = \sum_{k=0}^{N+L} |ep(k)|^2.
\]

But, using (16), \( ep(k) \) may be written as

\[
ep(k) = x(k) + \alpha_1 x(k-1) + \ldots + \alpha_L x(k-L).
\]

Or, \( ep(k) \) is one element of the the sequence \( ep(n) \)

\[
ep(n) = \sum_{i} x(i) \alpha_{n-i} = x(n) * \alpha_n.
\]

The symbol * denotes linear convolution, the sequence \( \alpha_n \) contains the prediction coefficients, with \( \alpha_0=1 \), and the sequence \( ep(n) \) has \( N+L+1 \) elements, that is \( ep(0), ep(1), \ldots, ep(N+L) \).

By using the Z-transform in (20), one obtains

\[
E(z) = X(z) \cdot A(z),
\]

where \( A(z) \) is the inverse filter polynomial, and its roots are inside the unit circle \(|z|=1\) (it is also named minimum-phase polynomial),

\[
A(z) = 1 + \alpha_1 z^{-1} + \ldots + \alpha_L z^{-L}.
\]

\( E(z) \) and \( X(z) \) are the Z-transform of \( ep(n) \) and \( x(n) \). It can be shown [4] that

\[
\sum_{k=0}^{N+L} |ep(k)|^2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} |E(e^{j\omega})|^2 d\omega =
\]

\[
= \frac{1}{2\pi} \int_{-\pi}^{\pi} |X(e^{j\omega}) A(e^{j\omega})|^2 d\omega.
\]

It is known that if the prediction order \( L \) is sufficiently high, then the expression \( Gp/1 \) \( A(e^{j\omega}) \) models the spectrum envelope of the sequence \( x(n) \), where the gain \( Gp \) is the square root of \( Ep \).

2.2. In frequency domain

Let \( s_1(t) \) a real signal, defined by (12). It is desired to compute a signal \( h(t) \),

\[
h(t) = h_0 + h_1 e^{j\Omega} + \ldots + h_L e^{jL\Omega},
\]

with \( h_0=1 \), such as the error signal \( e(t) = s_2(t) \) \( h(t) \) have the minimum energy:

\[
E = \frac{1}{T} \int_{0}^{T} |e(t)|^2 dt.
\]

For this reason \( h(t) \) is named the inverse signal. Let the next equation, that is demonstrated in [7]
\[
E = \frac{1}{T} \int_0^T |e(t)|^2 \, dt = \sum_{k=-N}^{N+L} |g_k|^2 ,
\]
where \( g_k \) is one of the elements of sequence
\[
g_n = h_n * b_n = \sum_i h_i b_{n-i} ,
\]
and \( b_n \) is the sequence with coefficients \( h_n, i=0, \ldots, n \) and \( b_n \) contains the Fourier coefficients of \( s(t), b_N, \ldots, b_N \).

Due to equation (27), \( g_k \) can be expressed as
\[
g_k = b_k + h_k b_{k-1} + \ldots + h_L b_{k-L} = b_k - (-h_1 b_{k-1} - \ldots - h_L b_{k-L}) = b_k - \tilde{b}_k .
\]

From (28) it follows that computing of \( h(t) \) is equivalent to a linear prediction for Fourier coefficients \( b_k \) of signal \( s(t) \). This method is called linear prediction in spectral (or frequency) domain (LPSD). The similarity between equations (23) and (26) (where \( E(e^{j\omega a}) \) corresponds with \( e(t) \), and \( e^{j\omega k} \) with \( g_k \), or (17) and (28), confirms the above statement.

Further, if \( Gp/1 A(e^{j\omega t}) \) represents the envelope of \( X(e^{j\omega a}) \), then \( Glbh(t) \) represents the envelope of \( s(t) \), where \( G \) is square root of \( E \).

The LPSD algorithm can also be applied on complex signals, as will be shown in the next section. In this case, equations (13), (14) and (15) can be used.

3. MinP-AllP Decomposition of a bandpass signal

Let \( s(t) \) expressed by
\[
s(t) = e^{j\omega t} \sum_{k=0}^N b_k e^{j\Omega t} ,
\]
where \( \omega_t = K \Omega, K < N \), that is, the signal \( s(t) \) is a bandpass signal.

If \( \xi = e^{-j\Omega t} \) is a complex-time variable, \( s(t) \) can be written as
\[
S(\xi) = \xi^{-K} (b_0 + b_1 \xi^{-1} + \ldots + b_N \xi^{-N}) = \\
= \xi^{-K-N} (b_0 \xi^{-N} + b_1 \xi^{-N-1} + \ldots + b_N) = \\
= \xi^{-K-N} b_0 (\xi - r_1)(\xi - r_2) \ldots (\xi - r_N) = \\
= \xi^{-K} b_0 (1 - r_1 \xi^{-1})(1 - r_2 \xi^{-1}) \ldots (1 - r_N \xi^{-1})
\]
where \( r_i, i=1, \ldots, N \), are the roots (zeros) of \( S(\xi) \). Then, \( s(t) \) can be obtained as \( S(e^{-j\Omega t}) \), or,
\[
s(t) = b_0 e^{j\omega t} \prod_{i=1}^N (1 - \xi^{-1}) = \\
= b_0 e^{j\omega t} \prod_{i=1}^P (1 - p_i e^{j\Omega t}) \prod_{i=1}^Q (1 - q_i e^{j\Omega t}) ,
\]
where \( p_i \) are zeros inside the unit circle and \( q_i \) are zeros outside the unit circle, and \( P + Q = N \).

By borrowing the terminology of FIR filters [6], the first product from (29) can be denoted as \( s_{MinP}(t) \) (minimum-phase signal), and the second product as \( s_{AllP}(t) \) (maximum-phase signal).

Next, the zeros from outside the unit circle will be reflected inside the unit circle (as \( 1/q_i \)), and then, \( s(t) \) will be divided with the same quantity,
\[
s(t) = b_0 \prod_{i=1}^P (1 - p_i e^{j\Omega t}) \prod_{i=1}^Q (1 - (1 - 1/q_i e^{j\Omega t})).
\]

The zeros inside the unit circle are grouped together, and thus is obtained a different minimum-phase signal, denoted as \( s_{MinP}(t) \). The zeros outside the circle and the poles that have been obtained by their reflections form an all-pass phase, \( s_{AllP}(t) \), by analogy with the terminology from FIR (all-pass system). The term \( e^{j\omega t} \) has unity magnitude, and the remaining numerator and denominator factors have the magnitude \( \prod_{i=1}^Q |q_i| \), that is, the magnitude of \( s_{AllP}(t) \) signal is time independent.

Next, based on (29) and (30) another form of \( s(t) \) can be obtained. Thus, it is known that for a minimum-phase signal, its phase is the Hilbert transform of its log envelope [6]. Thus, if \( e^{j\alpha(t)} \) is the envelope or the magnitude of a minimum-phase signal, its phase is \( \hat{\alpha}(t) \), and the signal can be expressed as \( e^{j(\alpha(t) + \hat{\beta}(t))} \). Similarly, a maximum-phase signal can be expressed as \( e^{j\beta(t) - \hat{\beta}(t)} \). Thus, \( s(t) \) can be written as (see [1])
\[
s(t) = A_c e^{j\omega t} e^{j(\alpha(t) + \hat{\beta}(t))} e^{j\beta(t) - \hat{\beta}(t)} ,
\]
where \( \omega_c = \omega_t + Q \Omega \), and \( A_c = b_0 \prod_{i=1}^Q |q_i| \). The expressions for \( \alpha \) and \( \beta \) are given also in [1].

In order to express \( s(t) \) on the form \( MinP(t)AllP(t) \), (31) will be multiplied and divided by \( e^{j\hat{\beta}(t)} \):
\[
s(t) = e^{j(\alpha(t) + \beta(t))} e^{j(\hat{\alpha}(t) + \hat{\beta}(t))} A_c e^{j(\omega_t - \hat{\beta}(t))} .
\]

It can be seen that the magnitude of \( AllP(t) \) is \( A_c \) and its phase is \( \omega_t - \hat{\beta}(t) \). Also, the log magnitude of \( MinP(t) \) is \( \alpha(t) + \hat{\beta}(t) \), and its phase is \( \hat{\alpha}(t) + \hat{\beta}(t) \). It follows that the log magnitude and phase contain the same information, and hence only one of them is
necessary, for instance, the log magnitude (the envelope).

Further, the log magnitude of \( s_1(t) \), without \( A_1 \), is \( \alpha(t) + b(t) \), that is, the same as log magnitude of \( \text{MinP}(t) \), and its phase is \( \alpha_1 + \hat{\alpha}(t) - \beta(t) \). The IF of \( s_1(t) \), i.e. \( \frac{1}{2\pi} \left( \alpha(t) + \hat{\alpha}(t) - \beta(t) \right) \), may take negative values, which are not physically justifiable. But, it can be seen that, by removing the Hilbert of log magnitude of \( s_1(t) \) from its phase, the phase of \( \text{AllP}(t) \) is obtained.

The IF of \( \text{AllP}(t) \) is \( \frac{1}{2\pi} (\alpha(t) - \hat{\beta}(t)) \), and as for the group delay of all-pass systems, it can be shown that it is always positive, even if \( \alpha_1 = 0 \), that is, this decomposition can be applied also for signals which are not of bandpass type. Thus the IF of \( \text{AllP}(t) \) component is named PIF (positive IF).

In conclusion, if \( s_1(t) \) can be decomposed in \( \text{MinP}(t) \) and \( \text{AllP}(t) \) components, the envelope of \( \text{MinP}(t) \) is exactly the envelope of \( s_1(t) \) (without \( A_1 \)), and the IF of \( \text{AllP}(t) \) will have only positive values. It can be demonstrated that \( A_1 \) is the gain \( G \) from linear prediction.

One of the ways to achieve this decomposition is to calculate the roots of \( s_1(t) \). It can be pointed out that this can be done easier by using the LPSD algorithm.

Thus, as previously shown, the LPSD algorithm computes \( h(t) \) in order to approximate the envelope of \( s_1(t) \), that is, \( 1/|h(t)| = |s_1(t)| \). It follows that \( |h(t)| = e^{-\alpha(t) - \beta(t)} \). Further, \( h(t) \) is a minimum-phase signal (because \( A(z) \) is a minimum-phase filter in linear prediction in time domain). The significance of this property is that the phase of \( h(t) \) is the Hilbert transform of its log envelope. Thus, \( h(t) \) will be

\[
h(t) = e^{-\alpha(t) - \beta(t)} e^{-j(\tilde{\alpha}(t) + \tilde{\beta}(t))},
\]

that is, \( 1/|h(t)| \) is the approximation to the \( \text{MinP}(t) \) component of \( s_1(t) \).

Therefore, the error signal \( e(t) \) will be \( e(t) = s_1(t) - h(t) \), where \( s_1(t) \) is given by (32) and \( h(t) \) is given by (33),

\[
e(t) = A_e e^{-j(\tilde{\alpha}(t) - 2\tilde{\beta}(t))},
\]

that is, \( e(t) \) is the approximation to the \( \text{AllP}(t) \) component of \( s_1(t) \).

Then, in order to obtain the PIF, the expression

\[
\frac{1}{2\pi} \int d(\text{angle} e(t)) dt
\]

must be computed. This can be done by computing the phase difference between neighboring samples, and then dividing it by sample time, \( T_e \),

\[
\phi(t) = \frac{1}{2\pi} \frac{\text{angle} (e(t + T_e)) - \text{angle} e(t)}{T_e},
\]

Next, two possibilities for LPSD implementation will be presented.

Let \( s_2(t) \) a real bandpass signal, that is in (12), some coefficients will be zero, \( a_{-K} = \ldots = a_{K} = 0 \). Then, if \( L \leq 2K-1 \), the energy of the error signal (see equation (26)) can be written as:

\[
E = \frac{1}{T} \sum_{k=-N}^{K} |g_k|^2 + \frac{1}{T} \sum_{k=K}^{N+L} |g_k|^2,
\]

or,

\[
E = \frac{1}{T} \int_0^T (|s_2(t) - \tilde{s}_2(t)| h(t))^2 dt + \frac{1}{T} \int_0^T (|s_2(t) + \tilde{s}_2(t)| h(t))^2 dt.
\]

The condition \( L \leq 2K-1 \) is necessary so as the two terms in the above expressions do not overlap. Because the analytic and antianalytic signals are complex conjugate, the two terms are equal:

\[
E = E_1 + E_2 = 2E_1.
\]

It follows that, by minimizing any one of the terms of (36) or (37), the same coefficients \( h_i \) are obtained.

Thus, in order to implement LPSD, there are two cases:

1. A real signal can be used. In this case one minimizes the first form of energy (26), but the condition \( L \leq 2K-1 \) must be achieved. The gain is \( G = \sqrt{E} = \sqrt{2E_1} \).

2. Also, a complex signal can be used (an analytic signal). In this case one minimizes one of the terms of (36) or (37), and the condition \( L \leq 2K-1 \) is not necessary. The gain is \( G_1 = \sqrt{E_1} = G / \sqrt{2} \).

Let \( s[n] \), \( n = 0, 1, \ldots, Q \) the samples of a real signal. The sampled version of \( e(t) \) will be

\[
e[n] = s[n] + \sum_{k=1}^{L} h_k s[n-k] e^{jk\Delta n}.
\]

Then \( \sum_{n=0}^{Q} |e[n]|^2 \) will be minimized, and the coefficients \( h_i \) will be obtained as the solution of a matrix equation [4].

If a complex signal is used in LPSD, then the autocorrelation method may be applied, where the input contains the Fourier coefficients, \( b_k \), \( k = 1, \ldots, N \), and the output contains the \( h_i \) coefficients of the inverse signal \( h(t) \), \( t = 0, 1, \ldots, L \).

If the analyzed signal is a speech signal, it is given only by its real-valued samples. Therefore, in order to apply the LPSD algorithm the condition \( L \leq 2K-1 \) must be met. For this, first the speech signal must be bandpass filtered. Next, the LPSD algorithm is applied by minimizing \( \sum_{n=0}^{Q} |e[n]|^2 \). Finally, for approximating
AllP(t) component by s_1(t)h(t), s_1(t) must be a complex signal. Therefore, the analytic signal will be computed for the speech signal, and then, the PIF will be computed using (35).

4. Experimental results

This section presents two examples to illustrate the theoretical principles from previous sections. The first example uses a complex signal known by its Fourier coefficients, and the second example uses a speech signal. The simulation was done in MATLAB.

Let the complex signal s_1(t) given by (8), with the following coefficients b_i, i=1,…7: 1.000; -0.6994 -3.2588i; -5.8397 + 0.7759i; 0.4721 + 7.3669i; 6.7262 - 2.0615i; -5.2124 - 7.1070i; 1.5334 + 2.9897i. The fundamental frequency was 62.5 Hz (Ω=2π/16 rad/ms), the sampling frequency was 16 kHz. The number of samples was 256, such as exactly 1 period of the signal was analyzed. The prediction order was chosen L=14.

First, for the real part of the presented signal, the analytic signal was computed using (6) and (7), and its imaginary part was identical with the imaginary part of the presented signal, so as expected. Fig. 1 presents the real part of the signal and its true envelope, computed as magnitude of the presented complex signal.

Fig. 2 presents the true envelope (solid line) along with estimated envelope (1/|h(t)|, dashed line). It can be shown that they are close. The LPSD algorithm was achieved using the autocorrelation method, by lpc function from MATLAB.

Fig. 3 presents the MinP(t) component of s_1(t) (dashed line) along with its envelope (solid line) and fig. 4 presents AllP(t) component of s_1(t). These components were obtained using the roots of the polynomial (P=2 zeros inside unit circle, and Q=4 zeros outside the unit circle) attached with s_1(t). It can be shown that the envelope of MinP(t) is the same with that of s_1(t), except for a scale factor. This scale factor is exactly the constant magnitude (about of 8.41) of AllP(t) component.

Fig. 5 presents three versions of IF of the signal s_1(t). The first, represented by solid line is computed by equation (35) applied for the s_1(t), and has values both negative and positive. The second, represented by dashed line represents the true PIF (positive IF) and is computed by equation (35) applied for AllP(t) component obtained by computing the roots of polynomial. The third, represented by dashed-dot line represents the estimated PIF, and is computed by applying (35) to signal e(t). The mean value of the
The estimated PIF is obtained as 250.027 Hz. This value is true, because PIF is \( \omega_t - 2 \hat{\beta}(t) \) and \( \omega_t = Q \Omega = 4 \cdot 2\pi \times 62.5 \text{ rad/s} \).

If the analysed signal is a real-valued bandpass one, it can be modeled by equation (12). It has the same coefficients with the previous signal, and the lower edge-band is \( K \Omega \) with \( K=9 \). The prediction order is also \( L = 14 \), thus the condition \( 2K-1 \geq L \) will be met. The LPSD algorithm is achieved by both versions, namely autocorrelation method (for a complex signal) and that based on matrix equation (for real signal, [4]). The envelope obtained is the same for both methods and the same with that for the previous signal. The autocorrelation method leads to the same results for the two signals (real and complex), because the input is the same in both cases, namely the Fourier coefficients.

Fig. 6 presents the signal and its envelope.

The signal which will be analysed next is a speech signal, namely a waveform of 12000 samples, with sampling frequency of 8 kHz. It corresponds to spoken utterance of the Romanian word “trei”. Then, samples 4001 … 4400 will be considered.

Fig. 7 and fig. 8 present the speech signal and its spectrum. The speech signal is first band-pass filtered by a 50-th order FIR filter with a 3 dB bandwidth of 600 Hz, centered around 1500 Hz. Fig. 9 and fig. 10 present the speech signal and its spectrum after filtering. Also, in fig. 9 is presented the true envelope of speech signal, obtained as \( s_a(t) \), where \( s_a(t) \) represents the analytic signal formed based on speech signal.

Fig. 5. Three versions of IF

Figure 5. Three versions of IF

Figure 6. The real signal and its envelope

Figure 6. The real signal and its envelope

Figure 7. The speech signal

Figure 8. The spectrum of speech signal

Figure 9. The filtered speech signal and its true envelope

Figure 9. The filtered speech signal and its true envelope
Next the LPSD algorithm is applied in order to obtain the inverse signal $h(t)$, and then, the error signal is $e(t) = s(t) - h(t)$ is computed. The error signal is needed to compute the PIF. The prediction order was $L = 10$.

Fig. 11 presents the speech signal and its estimated $(1/|h(t)|)$ envelope. It can be shown that there are differences compared with true envelope, but the approximation is good enough. Fig. 12 presents the PIF of speech signal. It has only positive values, except for a few values at the beginning of the interval, where the signal is zero, due to the delay of the FIR filter.

5. Conclusion

The paper presents several methods for obtaining two instantaneous components of a signal, the envelope and the instantaneous frequency. These methods have been applied first to a synthesized signal and then to a speech signal. The paper contains experimental results that confirm the theoretical statements.

6. References