Convergence Improvement for Oversampled Subband Adaptive Noise and Echo Cancellation

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Abstract

The convergence rate of the Least Mean Square (LMS) algorithm is dependent on the eigenvalue distribution of the reference input correlation matrix. When adaptive filters are employed in low-delay over-sampled subband structures, colored subband signals considerably decelerate the convergence speed. Here, we propose and implement two promising techniques for improving the convergence rate based on: 1) Spectral emphasis and 2) Decimation of the subband signals. We analyze the effects of the proposed methods based on theoretical relationships between eigenvalue distribution and convergence characteristics. We also propose a combined decimation and spectral emphasis whitening technique that exploits the advantages of both methods to dramatically improve the convergence rate. Moreover, through decimation the combined whitening approach reduces the overall computation cost compared to subband LMS with no pre-processing. Presented theoretical and simulation results confirm the effectiveness of the proposed convergence improvement methods.

1. Introduction

Subband Adaptive Filters (SAF) have become a viable choice for adaptive noise and echo cancellation. The SAF approach uses a filterbank to split the fullband input into a number of frequency bands, each serving as input to an adaptive filter. Due to their narrower bandwidth, subband signals can be decimated. Subband decomposition and decimation results in much "whiter" signals at the input of adaptive filter. Decimation also leads to a parallel bank of much shorter adaptive filters improving convergence behavior [1]. If critical sampling is used, aliasing distortion may be eliminated by employing either adaptive cross-filters between adjacent subbands or gap filters [1-2]. Systems with cross-filters generally converge slower and have higher computational cost, while systems employing gap filters produce significant signal distortion.

Over-Sampled SAF (OS-SAF) systems offer a simplified structure that, without employing cross-filters or gap filters, significantly reduces the aliasing level in subbands. Typically, in an attempt to reduce computation cost, a non-integer over-sampling factor close to one ($1 < \mathrm{OS} < 2$) is used (see for example [3]). However, for many real-time applications that require low processing delay, long analysis time-windows can not be employed. Consequently, high over-sampling factors are used to minimize the aliasing distortion that would occur in critical sampling or low over-sampling cases. As a result, prototype filter design becomes less stringent [4]. In adaptive noise and echo cancellation (and many other applications) wide-range gain adjustment of the subband signals is a necessity. To avoid aliasing and distortions, the solution of choice is to use over-sampling factors of 2 or more. However, over-sampling degrades the convergence behavior of SAF systems. In this research, we investigate the convergence properties of an OS-SAF system based on a generalized DFT (GDFT) highly over-sampled filterbank (OS = 2 or more).

We introduce the employed SAF system in Section 2. Section 3 provides the theoretical basis for eigenvalue analysis of the convergence problem. In Section 4, techniques for improving convergence of LMS-based adaptive algorithms are proposed and analyzed. Finally, system evaluations and conclusions are described in Sections 5 and 6.

2. SAF System Structure

Fig. 1 shows an SAF system in a noise cancellation application. Due to its desirable properties, a very efficient Weighted Overlap-Add (WOLA) filterbank [4] is employed in this research. The WOLA is a highly over-sampled GDFT uniform filterbank implemented on an ultra-low-power hardware platform. Through the DFT, it modulates a single prototype filter into K complex filters ($K/2$ real bands due to frequency symmetry). In order to take advantage of the over-sampling properties of the WOLA structure, we configure it for $R = 8$, so the over-sampling factor will be $\mathrm{OS} = K/R = 4$. The added computation cost due to over-sampling is partly compensated by the use of shorter analysis prototype filters, and the efficiency of the WOLA hardware structure [4]. Referring to Fig. 1, each adaptive processing block is generally an adaptive filter that works on a specific frequency band thus modeling a narrow frequency band of the acoustic plant.

As shown in Fig. 1, for white reference noise, input signals of the adaptive filter are no longer white in spectrum and their 6-dB bandwidth is limited to π/OS ($B_{-6dB} = \pi/4$). This significantly degrades the convergence properties of the LMS algorithm compared to the critical sampling case, where all subband signals are almost white.

Although the over-sampled subband signals are not white, their spectra are colored in a predicable way and can therefore be modified by further processing to "whiten" them in order to increase the convergence rate. This motivates us to propose and investigate spectral whitening techniques for OS-SAF systems. Since real-time low-delay and low-cost applications are targeted, the whitening techniques must be computationally efficient and should not substantially increase the system delay.

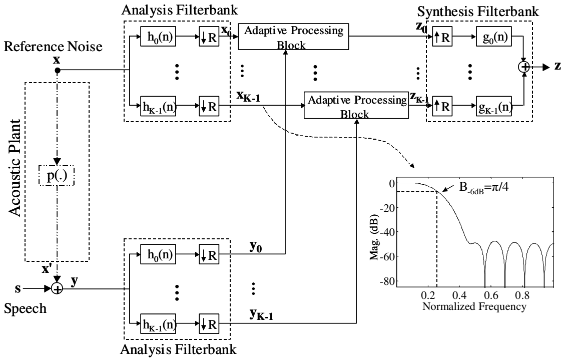

Fig. 1: Block diagram of SAF system

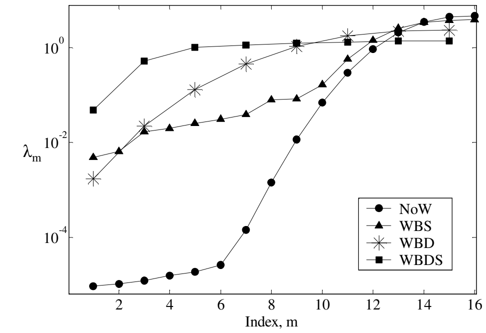

Fig. 2: Eigenvalues of the input correlation matrix in NoW, WBS, WBD and WBDS cases.

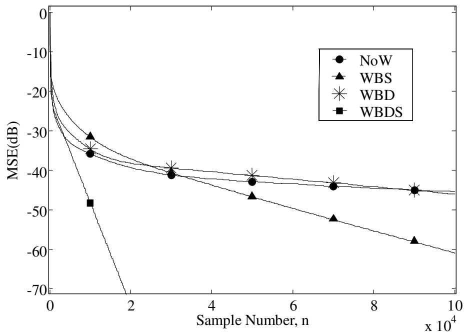

Fig. 3: MSE time-variations for NoW, WBS, WBD and WBDS cases.

3. Eigenvalue Analysis

The convergence behavior of LMS-based adaptive filters has been extensively studied in the literature [5]. In [6], Morgan has carefully studied convergence characteristics based on eigenvalues of the input correlation matrix. Assuming reference input vector $\mathbf{x}_n$, noisy signal sample y_n and filter weights vector $\mathbf{w}_n$ (of length M), the enhanced (noise cancelled) output sample is $z_n = y_n - \mathbf{w}_n^T\mathbf{x}_n$. For simplicity, subscript k representing the k[th] subband has been dropped. Morgan has shown that for moderate or high values of M (e.g., $M \geq 16$), a Modal Power Approximation (MPA) for Mean Squared Error (MSE) can be expressed as [6]:

$$\xi_n = E\{z_n^2\} = \xi_{\min} + \frac{1}{M}\sum_{m=1}^{M}\lambda_m(1-\mu\lambda_m)^{2n} \qquad (1)$$

where $\xi_{\min} = E\{s_n^2\} + E\{x_n'^2\} - \mathbf{r}_{xx'}^T\mathbf{R}_{xx}^{-1}\mathbf{r}_{xx'}$, $\mathbf{R}_{xx} = E\{\mathbf{x}_n\mathbf{x}_n^T\}$ and $\mathbf{r}_{xx'} = E\{\mathbf{x}_n x_n'\}$. s_n and x_n' are the speech sample and the noise component in the second (noisy) input. $\lambda_m, m = 1,...,M$ are the eigenvalues of correlation matrix $\mathbf{R}_{xx}$. We assume that the LMS step size is $\mu = 1.0/M$ and that the signal power is unity so that $\sum_{m=1}^{M}\lambda_m = \mathrm{tr}(\mathbf{R}_{xx}) = M$ and $\xi_0 - \xi_{\min} = 1$ for $n = 0$ in (1) (as suggested in [6]).

In this research, we extend the use of the MPA to characterize the expected (theoretical) convergence properties of OS-SAF systems. Since the WOLA filterbank provides nearly orthogonal subbands [4], only the first subband of the filterbank is considered for theoretical analysis. The total (fullband) MSE can be approximated as the sum of subband MSE's.

With a white noise excitation at the input of the SAF system, x_n will be a bandlimited white noise with a bandwidth of $B_{-6dB} = \pi/4$ for $OS = 4$. Assuming an adaptive filter $\mathbf{w}_n$ of length $M = 16$, the eigenvalues of $\mathbf{R}_{xx}$ are computed and plotted in Fig. 2 (no whitening (NoW) case) in ascending order. The MSE values associated with these eigenvalues calculated by (1) are plotted in Fig. 3. Although only the total distribution of eigenvalues can fully characterize the convergence behavior, it is obvious that the summand in (1) is dominated at the beginning (in time) by the terms due to the largest eigenvalues, and at the end by those due to the smallest eigenvalues. In other words, an increase in the largest eigenvalues results in a smaller MSE at the beginning of LMS adaptation, and an increase in smallest eigenvalues will accelerate final convergence.

4. Proposed Techniques for Whitening

To cope with the slow convergence problem of OS-SAF systems, we propose and implement two different whitening techniques and analyze their effects on the convergence rate of the LMS algorithm when employed in highly over-sampled SAFs.

4.1. Whitening by Spectral Emphasis

We have already reported the basics of this method in [7]. Here we further analyze the performance of the method and combine it with another method. Shown in Fig. 4 is the modified adaptive processing block diagram that employs whitening by spectral emphasis (WBS) for convergence improvement. Considering the subband signal spectrum, we have designed and employed a filter $f_{emp}(.)$ that amplifies the high three quarters of the spectrum (in each subband) and leaves the low quarter intact. The emphasized signals are used only to improve the convergence characteristics of the adaptive filter. As shown in Fig. 4, the adaptive filtering is done in the main branch (right branch) while the side branch does the spectral emphasis and LMS weight adaptation. In each iteration, the

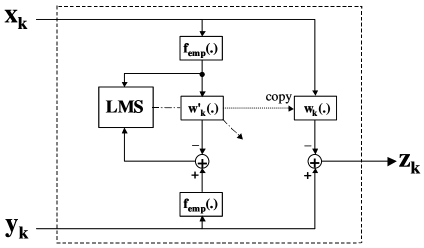

Fig. 4: Adaptive processing block in WBS method.

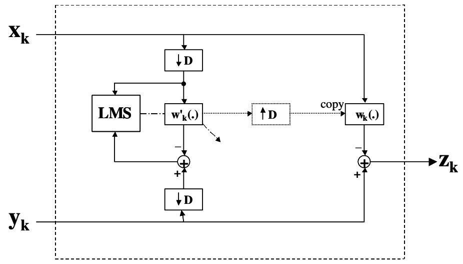

Fig. 5: Adaptive processing block in WBD method.

updated weights are copied from the side branch to the main branch. The use of spectral emphasis has no effect on the modeling behavior of the adaptive filter. Since spectral emphasis has been applied to both reference and noisy inputs, its effect will be cancelled out [8].

The spectral emphasis basically improves the convergence rate through amplification of small eigenvalues. Due to the unity gain constraint, this will cause the attenuation of larger eigenvalues. The eigenvalues of the correlation matrix of the emphasized reference signal are plotted in Fig. 2 in comparison with those for no whitening (NoW) case. As shown, WBS amplifies the smallest eigenvalues that can finally lead to faster convergence of the LMS algorithm.

The theoretical MSE time-variation is calculated (using (1)) for the eigenvalues of the emphasized reference signal and plotted in Fig. 3. Comparing with the NoW case, although attenuation of large eigenvalues has increased the MSE at the beginning of the adaptation, the boosted smallest eigenvalues later result in a much faster convergence rate in the WBS case.

Spectral emphasis improves LMS convergence at a cost of extra computations of spectral emphasis filtering. In the following section, another whitening technique is introduced that will accelerate convergence rate while decreasing the computation cost.

4.2. Whitening by Decimation

Since the subband reference signal has a limited bandwidth of $B_{-6dB} = \pi/4$ (for $OS = 4$), we propose to further decimate the subband signals, consequently generating whiter signals that will ultimately increase the convergence rate. Fig. 5 shows the adaptive processing block that employs the whitening by decimation technique (for convenience, called WBD). In this research for $OS = 4$, $D = 2$ is employed. Greater values of D were not used to avoid inband aliasing. By imposing a constant time-memory constraint on the adaptive filters, the order of the side branch adaptive filter $\mathbf{w}'_k(n)$ is set to $M/2 = 8$. After each LMS weight update, adaptive filter coefficients are upsampled and copied to the mirror filter in the main branch. Although the upsampling creates in-band images in the filter spectrum, since the input signal does not contain significant energy for $\omega > \pi/4$, the filter spectral images do not contribute to distortion at the output.

Eigenvalues of the correlation matrix of decimated reference signal are plotted in Fig. 2. To show the eigenvalues on the same graph in comparison to other methods, eigenvalues for the decimated method are located in odd indices. Decimation causes expansion of the effective bandwidth resulting in relatively larger eigenvalues. As can be predicted from this eigenvalue distribution, at the beginning the MSE will be larger than in the NoW case. However, the effects of the large eigenvalues will diminish and the small eigenvalues will take over. This plot predicts that whitening by decimation will ultimately result in lower MSE values. This is demonstrated in Fig. 3.

WBD is also attractive from computation cost point of view. Here, LMS adaptation is done once out of each D input blocks (once every 2 blocks in the $D = 2$ case). Furthermore, the inserted zero weights of the adaptive filter $\mathbf{w}_k(n)$ will cause a further reduction in computational cost. This will be further discussed in Section 5.1.

4.3. Whitening by both Decimation and Spectral Emphasis

Whitening by decimation improves the convergence rate by increasing the effective bandwidth of the reference input. Nevertheless, it cannot deal with the stop-band region of the prototype filter. Thus as it is evident from Fig. 2, WBD still suffers from the effects of the smallest eigenvalues. This motivates us to propose the use of a spectral emphasis filter following the whitening by decimation method. We refer to this hybrid method as WBDS. The block diagram of the combined adaptive processor is shown in Fig. 6. A low-cost second-order IIR spectral emphasis filter has been employed here.

As shown in Fig. 2, WBDS results in a majority of large (and approximately equal) eigenvalues and a few smaller ones. A much whiter spectrum of input signal is predictable from this eigenvalue distribution. The square-symbol curves in Fig. 3 demonstrate the considerable achieved improvement in convergence rate. Although the smaller values of the largest eigenvalues (compared to other methods presented) increase the MSE at the beginning, this is soon compensated by improvements due to other eigenvalues and consequently, general convergence behavior is much better than other presented methods.

5. System Evaluation

5.1. Computation Cost Comparison

We compare the computation cost of LMS adaptation and adaptive filtering in one subband per input sample for various methods. Assuming an adaptive filter of length M, the NoW approach requires $2M + 1$ operations (OPS, defined here as one complex multiply and add). Considering a spectral emphasis filter of order P_1, the computation cost of WBS is is $3M + 2P_1 + 1$ OPS. For the WBD method with a decimation factor of D, a total of $2M/D^2 + (M+1)/D$ OPS are needed. Finally, the total complexity of the WBDS method with a

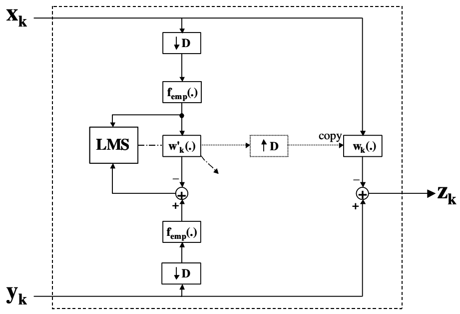

Fig. 6: Adaptive processing block in WBDS method.

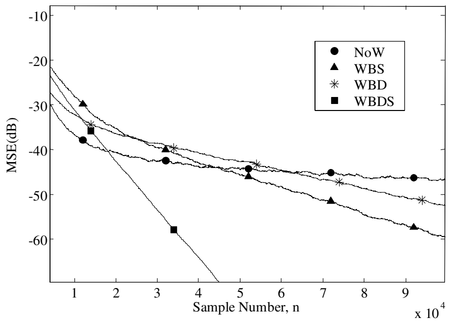

Fig. 7: Simulation results: MSE time-variations of the noise-cancelled first-subband output for NoW, WBS, WBD and WBDS cases.

spectral emphasis filter of order P_2 is $2M/D^2+(2M+2P_2+1)/D$ OPS. For a typical parameter setup ($M=16$, $D=2$, $P_1=4$ and $P_2=2$), the NoW, WBS, WBD and WBDS techniques require 33, 57, 17 and 19 OPS, respectively. This demonstrates a reduction by a factor of (almost) two in computation cost achieved by the use of the decimation technique (19 compared to 33 OPS). Moreover, in a real-time implementation, decimation allows the side branch adaptation process for different subbands to be spread across D consecutive blocks.

5.2. Convergence of Adaptive Noise Cancellation System

Convergence behavior of OS-SAF with and without the proposed whitening techniques has been evaluated by simulating the system in an adaptive noise cancellation setup. A white noise signal for the reference input and a typical acoustic plant are used. The WOLA filterbank has been configured for $K/2=16$ bands, $L=128$ point window length and down/up sampling ratio of $R=8$. So, the over-sampling ratio is $OS=4$. A 16-tap adaptive filter has been employed for each subband.

Fig. 7 presents the simulation results (MSE of the noise-cancelled first-subband output) for the NoW, WBS, WBD and WBDS cases. Despite all the simplifications and approximations made in our theoretical analysis, the simulation results consistently follow the corresponding theoretical curves shown in Fig. 3. As expected, the WBDS method has dramatically increased the convergence rate.

To ensure that the proposed techniques are not dependent on the transfer function of the acoustic plant, the simulations were repeated for several plants achieving consistent results. During the simulation, we also compared the MSE time-variations for the (full-band) time-domain outputs. Similar comparative results were achieved [8].

6. Conclusion

Based on a theoretical eigenvalue analysis of LMS convergence, we proposed, implemented, and carefully evaluated three simple and low-complexity whitening techniques that can significantly enhance the convergence rate of OS-SAFs.

It was shown that spectral emphasis whitening improves the convergence rate considerably but it requires extra computations to do spectral emphasis filtering. On the other hand, whitening by decimation achieves a data rate closer to critical sampling and provides a whiter signal at the adaptive filter input. By doing so, it actually decreases the computation cost. The combined and spectral emphasis whitening exploits the advantages of both methods and improves LMS convergence rate drastically.

Employing a side branch in the adaptive system enabled us to use the whitened inputs for the LMS adaptation (in the side branch) while conserving their original versions for the adaptive filtering in the main branch to avoid excessive aliasing. Our simulation results on an OS-SAF employed for noise cancellation confirm the capability of the proposed whitening techniques to effectively improve the convergence rate. We are now in the process of porting the OS-SAF system including the efficient WBDS technique to an ultra-low-power hardware implementation of the over-sampled WOLA filterbank described in [4].

7. References

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