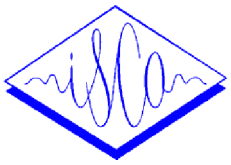

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ACTIVE BIOMEDICAL MEDIA EXPLORATION BY MEANS OF SPECTRAL ANALYSIS APPROACHES AND CHAOTIC SIGNAL ATTRACTOR TRAJECTORIES INVESTIGATION

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Abstract. The mathematical model of nervous pulse propagation in the homogeneous nervous fibre is constructed on the basis of spectral analysis methods. The results of digitized cardiosignal investigation on the basis of nonlinear dynamics approach (local - topological analysis of chaotic attractor trajectories) and spectral analysis methods (fast Fourier transforming) are represented. It is shown that the local - topological analysis of phase trajectories allows to estimate a number of freedom degrees of electrocardiosignal and complexity degree of myocardium dynamics.

Keywords: autowaves, nervous pulse propagation, minimal embedding dimension.

Introduction

Active media represent the basic substance of self-organization complex systems. The examples of active media are the working substance of a laser, gase plazma in technique; the heart muscle, eye retina, muscle and nervous fibres in medicine. The active media functionating is accompanied by propagation of autowaves representing a particular class of nonlinear waves, which spread in the active media at the expense of the energy stored in the medium [1,2]. Investigations of autowave processes allow to detect many new laws in complex systems behaviour and create on principle new technologies in signal and image processing. It has been shown that autowave methods employment allows to create highly parallel algorithms for pattern analysis which increase a computional speed from 3 to 6 orders of magnitude in comparison with a sequential (von Neumann) computer [3].

Complex systems (CS) behaviour is defined by included active media state. Most recent investigations on physiologic processes in medical CS such as heart rate, blood pressure, or nerve activity have shown that biomedical signals vary in a complex and irregular way, even during stable external conditions [4-5]. Chaotic dynamics of human organ systems offer many functional advantages. Chaotic systems operate under a wide range of conditions and are therefore adaptable and flexible. In recent years, considerable attention has been devoted to unifying various aspects of cardiac physiology using nonlinear dynamics, particularly through methods of topological analysis and fractal geometry [6]. In this paper chaotic dynamics of cardiac activity is investigated. We propose a method of estimating complexity degrees of heart dynamics by means of electrocardiogram (ECG) time series exploration on a base of local - topological analysis [7-8] of attractor constructed from electrocardiosignal. Recently the method has been suggested for a local-topological analysis of attractor trajectories constructed from chaotic time series (CTS) [7-8]. This method allows to reduce demanded experimental data quantity and spare computer resources (time, storage) in comparison with traditional algorithms [9, 10]. Besides, it provides a good convergence of computation process and obtaining reliable numerical results.

Mathematical modeling of signal propagation in the active media

A task of great importance is the mathematical modeling of signal propagation in the active media. Solving this problem is necessary for creating techniques of the control by separate parts of a complex system and for constructing new technologies in communication. In this paper we propose a method of modeling active media

excitement and signal propagation along the nervous fibre. The suggested method is based on spectral analysis algorithms (including fast Fourier transformation). The methods proposed in this paper provide essential reduction of computation complexity and high stability of the algorithms.

Let x be a coordinate in the nervous fibre, the pulse spreading along this coordinate. Then $u(x,t)$ is a potential difference of the excitement pulse. In accordance with Hodgkin - Hucksly model and FitzHugh - Nagumo approach [1, p.311-330], the nervous fibre as one-dimensional active medium is described by the following nonlinear equation:

$$\frac{\partial u}{\partial t} - D_u \frac{\partial^2 u}{\partial x^2} = -\frac{1}{C} i(u) \qquad (1)$$

where D_u is a diffusion coefficient equals to: $D_u = \frac{1}{RC}$; R and C are the resistance and capacity of a membrane respectively per unit length of the fibre; i is an ion current through the nervous fibre membrane that is approximated by a cubic parabola:

$$i(u) = B\,u\,(u-u_1)(u-u_2) \qquad (2)$$

In the formula (2) u_1 characterizes the beginning of a refractering period; u_2 is a difference between the characteristic voltages for Na current; B is normalizing factor. Cinetic models obtained with FitzHugh - Nagumo approach are often used when investigating autowave processes [11].

The results that are in a good accordance with experiments have been obtained with quazilinear time approximation of the ion current [12]. This approach can be considered as averaged cubic approximation. We consider the case when the pulse spreads along the homogeneous nervous fibre without distortions. In such a case the velocity v is constant and a "running" variable ξ can be introduced as follows [1,12]:

$$\xi = x - vt. \qquad (3)$$

Then the quazilinear approximation has a form:

$$i_I(\xi) = \begin{cases} i_{ex} & when\ -v\Delta t_{ex} \le \xi < 0, \\ i_r & when\ -v\tau \le \xi < -v\Delta t_{ex} \\ 0 & otherwise \end{cases} \qquad (4)$$

i_{ex} and i_r are the average values of the direct and inverse membrane currents respectively, τ is a pulse length, Δt_{ex} .is the length of excitement period. Evidently, this exchange of variables allows to make our consideration invariant over x. Inserting (3) into (1) we obtain for the voltage $\varphi(\xi)$ of the nervous pulse the following equation:

$$D_\varphi \frac{d^2\varphi}{d\xi^2} + v\frac{d\varphi}{d\xi} = \frac{1}{C} i_I(\xi) \qquad (5)$$

where $D_\varphi = D_u$. Further, we apply to (5) Fourier transformation:

$$\Phi(\omega) = \int_{-\infty}^{\infty} \varphi(\xi)\, e^{-j\omega\xi}\, d\xi \qquad (6)$$

and obtain the equation:

$$\omega(jv - \omega D_\varphi)\Phi(\omega) = \frac{j}{\omega C} f_2(\omega), \qquad (7)$$

where

$$f_2(\omega) = i_{ex}(1 - e^{j\omega v\Delta t_{ex}}) + i_r(e^{j\omega v\Delta t_{ex}} - e^{j\omega v\tau})$$

Consequently, the potential difference Fourier-image of nervous pulse $\varphi(\xi)$ can be expressed as:

$$\Phi(\omega) = \frac{j f_2(\omega)}{C\omega^2(jv - \omega D_\varphi)} \qquad (8)$$

From the expression (8) the values of real $\Phi_R(\omega)$ and imagine $\Phi_I(\omega)$ parts of nervous pulse spectrum can be obtained. Now one can use to $\Phi_R(\omega)$, $\Phi_I(\omega)$ any algorithm of fast inverse Fourier transformation [13] and obtain the form of the nervous pulse $\varphi(\xi)$ in any point of the

nervous fibre. In such approach the result is obtained without solving differential equation (1).

Electrocardiosignal exploration

Basic information obtained by experimental investigation of nonlinear dynamic systems (NDS) with the chaotic behaviour is CTS: $\xi(i)$, where i=1,2,...,N, Δt is the time interval of measurement. The behaviour of NDS can be completely described by means of construction of the chaotic attractor R_A^d in Euclidean space R^d. In accordance with Takens method [14], for m ≤ 2d+1 the points of the chaotic attractor $R_A^m \subset R^m$ are given by:

$$\vec{x}_j^{(m)} = (\xi(jp), \xi((j+1)p),...,\xi((j+m-1)p)), \quad (9)$$

where $j=1,2,...,L^{(m)}$; $L^{(m)} = N_p - m + 1$; m is embedding dimension, p is the delay interval. Takens method employment means construction of reduced CTS: $\xi_l = \xi(lp)$, where $l = 1,2,...,N_p$; and the value N_p is given by: $N_p \approx N/p$.

For the attractor construction we must use the minimal value of m, because in this case we reduce the value of calculations, experimental data quantity and measuring time. The minimal embedding dimension m_0 characterizes the upper number of NDS freedom degrees and the minimal number of differential equations demanded for mathematical modeling of NDS [15]. Consequently, value of m_0 is a characteristic of complexity degree of the investigated process and defines the physical state of NDS.

For computing m_0 many correlative-topological methods are used, among them Grassberger – Procaccia algorithm (GPA) [9] is most conventional. Such methods have large computation complexity and demand long CTS ($N \approx 10^4 - 10^5$; $N_p \approx 2\times10^3 - 2\times10^4$) for their implementation. Besides, it is a common feature of such algorithms that the quantity of required data points increases exponentially with m_0, resulting in exponentially growing requirements on measuring time, computer time, and data storage capacity [15]. Because of this fact the GPA and similar methods have been applied in most cases only to low-dimensional (typically $m_0 \le 10$) systems.

In this paper we propose a method of chaotic signals processing and m_0 determination based on R_A^m topological structure analysis. Our method requires much less experimental data quantity and is stable to changing m_0. The basic idea of our method is following. On the set of R_A^m we construct a function $Z(m)$, that defines a measure of topological instability of the attractor when enlarging phase space dimension ($R^m \to R^{m+1}$). The value of $Z(m)$ changes monotonously when enlarging m, but if $m \ge m_0$, then $Z(m) = Z_S$ and does not depend on m [7-8].

The topological structure of the chaotic attractor R_A^m can be defined by computing distances between neighbour points of R_A^m:

$$r_{j,j+1}^{(m)} = \left\| \vec{x}_j^{(m)} - \vec{x}_{j+1}^{(m)} \right\|, \quad (10)$$

where $j = 1,2,...,L^{(m)} - 1$. Distances (10) increase when $R^m \to R^{m+1}$, because from (9), (10):

$$r_{j,j+1}^{(m)} = \left[\sum_{l=0}^{m-1} (\xi_{j+l} - \xi_{j+l+1})^2 \right]^{\frac{1}{2}} \quad (11)$$

and

$$r_{j,j+1}^{(m+1)} = \left[\sum_{l=0}^{m} (\xi_{j+l} - \xi_{j+l+1})^2 \right]^{\frac{1}{2}} \ge r_{j,j+1}^{(m)}$$

So we suggest to describe changes of topological structure of R_A^m by relative distances between attractor points :

$$\beta_j^{(m)} = \frac{r_{j+1,j+2}^{(m)}}{r_{j,j+1}^{(m)}}, \quad (12)$$

where $j = 1,2,...,L^{(m)} - 2$.

Consequently, dynamics of changes in R_A^m topological structure when $R^m \to R^{m+1}$ can be represented by the sequence $\{\gamma_j^{(m,m+1)}\}$, the members of that being ratios of relative distances (12) in R^m and R^{m+1} respectively:

$$\gamma_j^{(m,m+1)} = \frac{\beta_j^{(m)}}{\beta_j^{(m+1)}}, \qquad (13)$$

where $j = 1,2,\ldots, L^{(m+1)} - 2$.

Further we average the members of the sequence $\{\gamma_j^{(m,m+1)}\}$ over all j and obtain the function of instability $Z(m)$ from the formula:

$$Z(m) = \frac{1}{L^{(m+1)} - 2} \sum_{j=1}^{L^{(m+1)}-2} \gamma_j^{(m,m+1)} \qquad (14)$$

The value $|Z(m) - Z_S|$ is a measure of topological instability of R_A^m when transformation $R^m \to R^{m+1}$. At least, we introduce the coefficient of topological stabilization in R_A^m as follows:

$$\rho(m) = \frac{Z(m+1)}{Z(m)} \qquad (15)$$

If the topological stabilization of R_A^m is happened (i. e. the topological structure of the chaotic attractor is invariant to transformation $R^m \to R^{m+1}$), we have that $m \geq m_0$, $Z(m) = Z_S$, where Z_S does not depend on m. Consequently if $m \geq m_0$, $\rho(m) = 1$ in this case, that is confirmed by results of numerical experiments (see tabl. 2).

For numerical implementation ECG time series with N_m=2500 length is investigated, Δt = 2 ms. In accordance with Kotelnikov theorem [16] a maximal frequency value of ECG spectrum equals to $f_m = \frac{1}{2\Delta t}$ = 250 Hz. Consequently, spectrum width is equal 500 Hz .For spectrum computation we take first N_{sp} points of CTS, $N_{sp}=2^{11}=2048$. Amplitude spectrum A_i, i=1,2,...,N_{sp} is computed by means of highly effective algorithm developed in [13] (program RSFFT).

Minimal frequency interval $\Delta f = \frac{2f_m}{N_{sp}}$=0.244 Hz. At the same time, boundary frequency value of analog-digital transformation device equals to f_b =120 - 125 Hz. Consequently only approximately N_b=500 points of obtained amplitude spectrum are reliable, that respects $f_b^c = N_b \Delta f$ = 122 Hz. The amplitude spectrum A_i with a boundary value $f_b^c/2$ with interval $2\Delta f$ (i. e. N_b /4 points) is represented in table 1. Our numerical results prove that heart activity dynamics is actually high irregular and displays chaotic behavior features. Corresponding curve of spectrum is displayed in fig. 1.

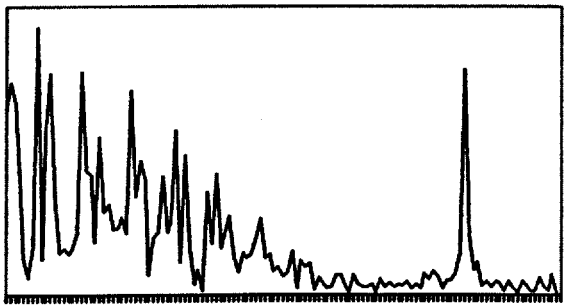

Fig. 1. ECG signal Fourier amplitude spectrum

As follows from table 1, the amplitude spectrum has some local maximum points near 50 Hz frequency (that corresponds to spectrum values with numbers about N_f=100 – 108. This is a result of the external influence of electric power with 50 Hz frequency. As follows from tabl.1 this kind of external noise doesn't distore ECG signal significantly because the common power of this noise is very small in comparison with common ECG signal power. Nevertheless, we filtered this noise by means of inverse Fourier transforming algorithm [13] and obtained ECG without this hindrance. The reduction of noise level in spectrum points A_{if} with numbers N_f is implemented as follows: $A_{if}^{(c)} = 10\sqrt{A_{if}}$.

Further, we investigate the topological stabilization of attractor constructed from ECG. For attractor construction Takens method has been used. In our numerical experiments N_p=200, $1 < p < 4$, N=200 - 800. In common ECG length equals to 2500 points and we used only the first part of it. Numerical implementation of local- topological analysis has been made over relationships (11)– (15). The values of coefficient of topological stabilization ρ(m) are represented in tabl.2. The numerical results confirm the convergence of used algorithm: the value of stabilization coefficient adopts a constant level when $m \geq m_0$. Consequently, the value of m_0 can be determined from the nature of dependence ρ(m), represented in the tabl.2.

Tabl. 1

N	A_i	N	A_i	N	A_i	N	A_i	N	A_i
1	7655	26	2817	51	3313	76	827	101	617
2	8842	27	3244	52	1635	77	302	102	896
3	7931	28	2586	53	925	78	131	103	1710
4	5565	29	8477	54	1733	79	803	104	9396
5	1458	30	4087	55	1560	80	456	105	2422
6	739	31	5611	56	1725	81	344	106	1043
7	1963	32	4774	57	2329	82	351	107	1368
8	11144	33	832	58	3162	83	432	108	381
9	1495	34	2354	59	1530	84	93	109	527
10	6872	35	2652	60	1686	85	662	110	314
11	9179	36	4907	61	963	86	308	111	567
12	3991	37	2633	62	1172	87	479	112	474
13	1727	38	3046	63	783	88	334	113	182
14	1923	39	6852	64	937	89	437	114	528
15	1706	40	1359	65	1782	90	365	115	277
16	1929	41	5806	66	248	91	571	116	122
17	2570	42	1959	67	1405	92	278	117	545
18	9286	43	463	68	1152	93	411	118	435
19	5128	44	1063	69	1308	94	272	119	176
20	4985	45	175	70	192	95	874	120	185
21	2212	46	4249	71	711	96	632	121	706
22	6595	47	2149	72	449	97	959	122	316
23	3446	48	5044	73	258	98	746	123	155
24	3798	49	1973	74	319	99	297	124	815
25	2751	50	2507	75	833	100	628	125	114

We consider that topological stabilization takes place (i.e. $m = m_0$) when $|\rho(m)-1|<\varepsilon$. From numerous numerical experiments [7] results we adopt that $\varepsilon=0.01$. From calculated dependence $\rho(m)$ one can see that $m_0=5$. This result is in a good coinciding with those obtained by other authors with GPA employment [4]. At the same time, CTS length in our investigations N=200 - 800, and in [4] N=16000, that is more than an order longer. Consequently, local-topological method allows to reduce experimental data quantity and spare computer resources. Calculations have been made with computer "Pentium" (tact frequency is 133 MHz, without mathematical coprocessor), computer time for calculating spectral characteristics (as well as one group of topological dependences) being less than 0.3 sec.

At least, we computed m_0 from various segments of ECS (at the middle and end). The results of m_0 computation are as well as in tabl.2

Tabl. 2

m	$\rho(m)$			
	p=1, N=200	p=2, N=400	p=3, N=600	p=4, N=800
1	0.001	0.000	0.553	0.002
2	0.885	0.894	0.946	0.936
3	0.960	0.987	0.990	0.985
4	0.983	0.985	0.987	0.981
5	0.994	0.996	0.999	0.992
6	0.994	0.997	0.996	0.996
7	0.998	0.997	0.998	1.000
8	0.999	1.000	1.000	0.998
9	0.999	0.999	0.998	0.999
10	0.998	0.998	1.000	0.999
11	1.000	1.000	0.999	0.999
12	1.000	0.999	0.999	1.000
13	1.000	1.000	1.000	0.999
14	0.999	0.999	0.999	0.999
15	1.000	1.000	1.000	0.999
16	1.000	1.000	1.000	0.999
17	1.000	1.000	1.000	1.000
18	1.000	1.000	1.000	1.000
19	1.000	1.000	1.000	1.000

(i.e. m_0=5), that one more time proves high reliability of local-topological analysis.

Consequently, this algorithm of the minimal embedding dimension determination essentially increases the efficiency of ECG investigation.

Conclusions

In this paper the mathematical model of nervous pulse propagation in the homogeneous nervous fibre is constructed on the basis of spectral analysis methods. Our method is more suitable for autowave propagation analysis in comparison with traditional that are based on the numerical solving of differential equations [1,17], because it has the following advantages.

1.The method is more robust, i.e. its structure does not depend on a choice of initial conditions and parameters of the active medium.

2.The algorithm suggested provides a good convergence of calculation process with reducing computer resources because there are no complex iteration circles in our method.

3.This method allows to work straightly with experimental data because the variables in (7), (8) can be measured in experiment.

Further, the heart activity dynamics has been investigated. It is shown that cardiac activity is a high irregular process with chaotic behavior features. The local-topological analysis of ECG time series has been made and its results allowed to estimate complexity degree of a cardiac process under investigation. The examination of heart dynamics complexity can provide important physiologic and prognostic information not detected by conventional methods of analysis.

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